Reaction mechanism of fusion-fission process in superheavy mass region

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Sochi, Russia, 5-11 September, 2011

1. Model

Coupled-channels method + Dynamical Langevin calculation

Trajectory analysis ← Langevin equation

Two center shell model

2. Results

$^{36}S+^{238}U$ and $^{30}Si+^{238}U$

Capture Cross-section

Fusion Cross-section

Mass distribution of Fission fragments

3. Mechanism of Dynamical process

Potential energy surface on scission line

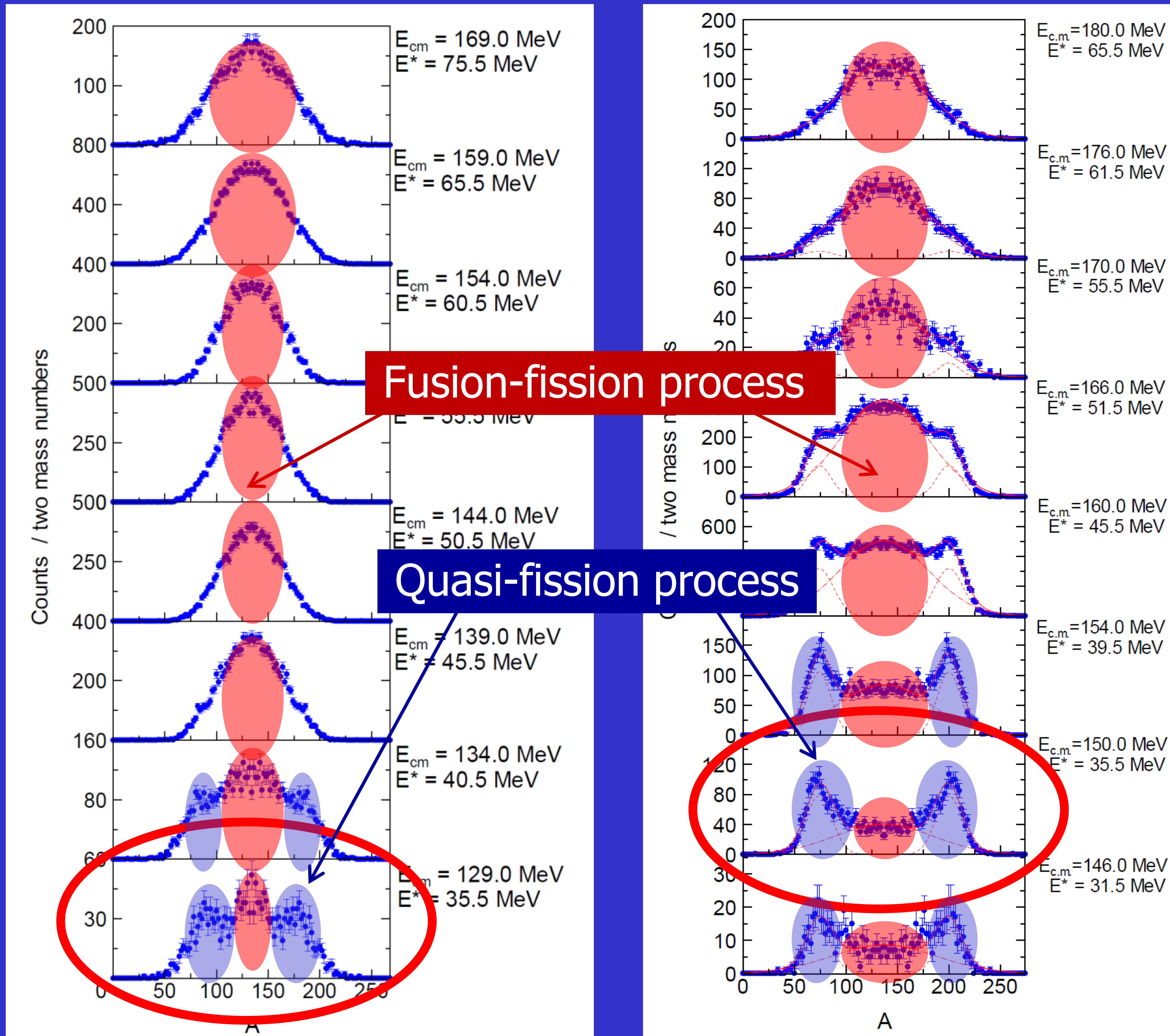
Analysis of trajectory behavior

Analysis of probability distribution

4. Summary

← Zcn=106 $^{30}Si + ^{238}U$ $^{36}S + ^{238}U$ Zcn=108 →

Exp. by K. Nishio *et al.* *at JAEA*



1. Model

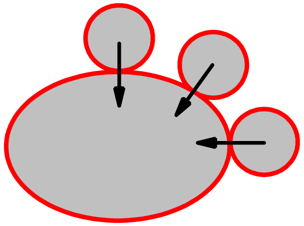
1-1. Estimation of cross sections

1-2. Dynamical Equation

Estimation of cross sections

Capture Cross Section

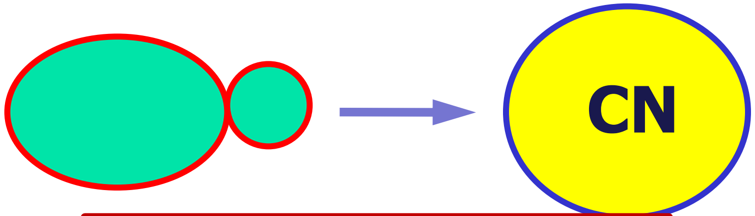
$$\sigma_{\rm cap}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm cap}(E;\theta),$$



$$\sigma_{\rm cap}(E;\theta) = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)T_\ell(E;\theta),$$

Coupled-channel method

Fusion Cross Section



Formation probability P_{CN}

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta),$$

$$\sigma_{\rm fus}(E;\theta) = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)T_\ell(E;\theta)P_{\rm CN}(E,\ell,\theta)$$

Dynamical calculation
Langevin eq.

Axial symmetric configuration

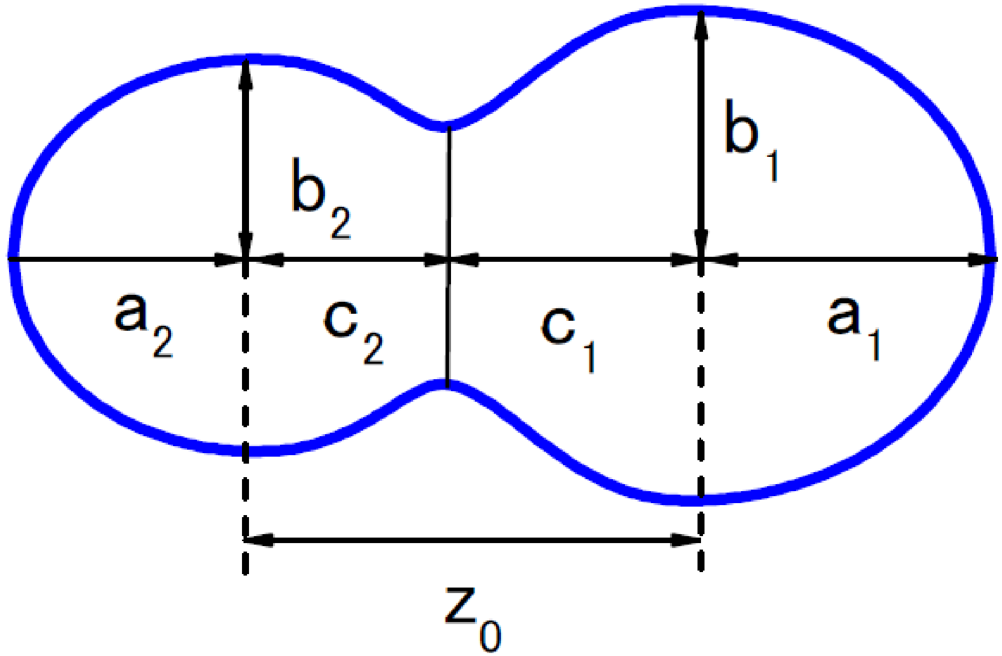
← deformation effect by assuming the angle dependent touching distance

Nuclear shape

two-center parametrization (z, δ, α)

(Maruhn and Greiner,
Z. Phys. 251(1972) 431)

$$q(z, \delta, \alpha)$$



$$z = \frac{z_0}{BR}$$

$$B = \frac{3+\delta}{3-2\delta}$$

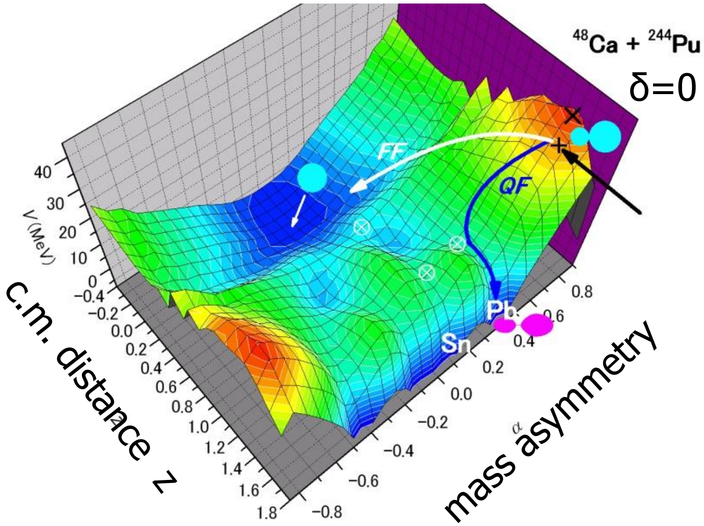
R : Radius of the spherical compound nucleus

$$\delta = \frac{3(a-b)}{2a+b}$$

($\delta 1 = \delta 2$)

$$\alpha = \frac{A_1 - A_2}{A_{CN}}$$

Trajectory which enters
into the spherical region
= fusion trajectory



Potential Energy

$$V(q,\ell,T) = V_{DM}(q) + \frac{\hbar^2 \ell(\ell+1)}{2I(q)} + V_{SH}(q,T)$$

$$V_{DM}(q) = E_S(q) + E_C(q)$$

$$V_{SH}(q,T) = E^0_{shell}(q)\Phi(T)$$

T : nuclear temperature

$E^* = aT^2$ $\quad a$: level density parameter

Toke and Swiatecki

E_S : Generalized surface energy (finite range effect)

E_C : Coulomb repulsion for diffused surface

$E^0{}_{shell}$: Shell correction energy at T=0

I : Moment of inertia for rigid body

$\Phi(T)$: Temperature dependent factor

$$\Phi(T) = \exp\left\{-\frac{aT^2}{E_d}\right\}$$

$$E_d = 20 \text{ MeV}$$

Multi-dimensional Langevin Equation

$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{jk} p_j p_k - \gamma_{ij}\left(m^{-1}\right)_{jk} p_k + g_{ij} R_j(t)$$

Friction dissipation — Random force fluctuation — Newton equation

$\langle R_i(t)\rangle = 0,\ \langle R_i(t_1)R_j(t_2)\rangle = 2\delta_{ij}\delta(t_1 - t_2)$: white noise (Markovian process)

$$\sum_k g_{ik} g_{jk} = T\gamma_{ij}$$

q_i : deformation coordinate (nuclear shape)

two-center parametrization (z, δ, α) (Maruhn and Greiner, Z. Phys. 251(1972) 431)

p_i : momentum

m_{ij} : Hydrodynamical mass (inertia mass)

γ_{ij}: Wall and Window (one-body) dissipation (friction)

$$E_{\text{int}} = E^* - \frac{1}{2}\left(m^{-1}\right)_{ij} p_i p_j - V(q)$$

E_{int} : intrinsic energy, E^* : excitation energy

2. Results

2-1. Capture and Fusion Cross sections

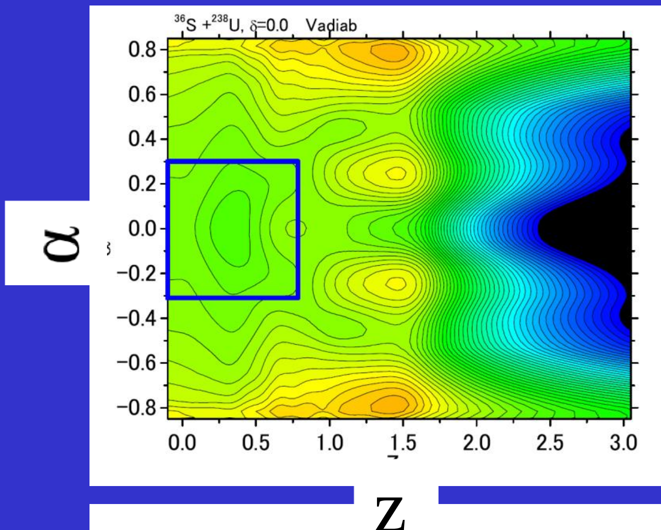
2-2. Mass distribution of Fission Fragments

$^{36}S+^{238}U$ and $^{30}Si+^{238}U$

Touching probability ← CC method

Perform a trajectory calculation
starting from the touching distance between target and projectile
to the end each process.

Fusion box and Sample trajectory $^{36}S + ^{238}U$



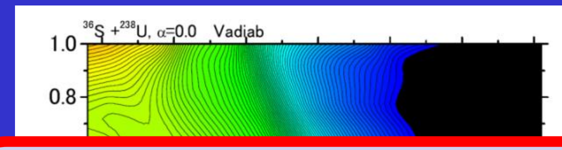
z-α

δ=0

E*= 40 MeV

L = 0

θ = 0

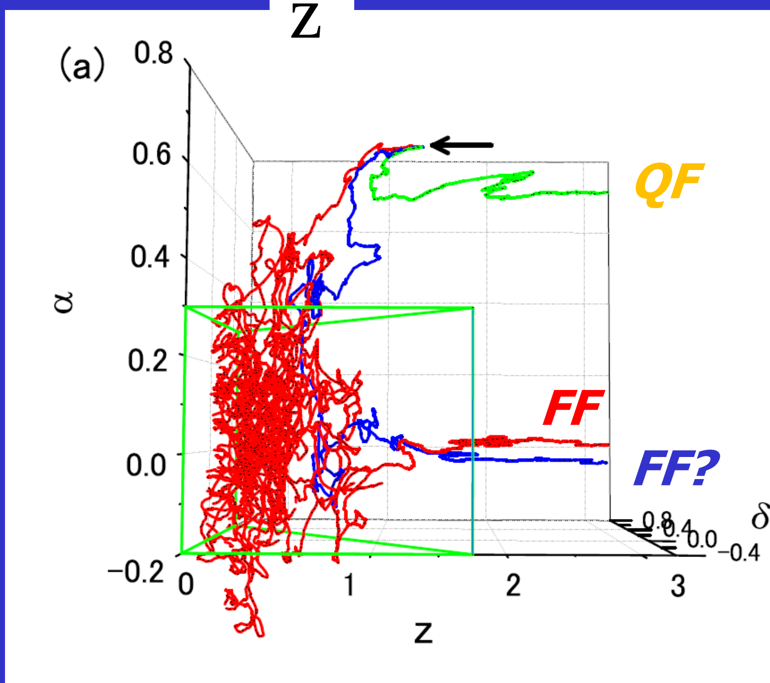


z-δ

DQF (Deep Quasi-fission process)

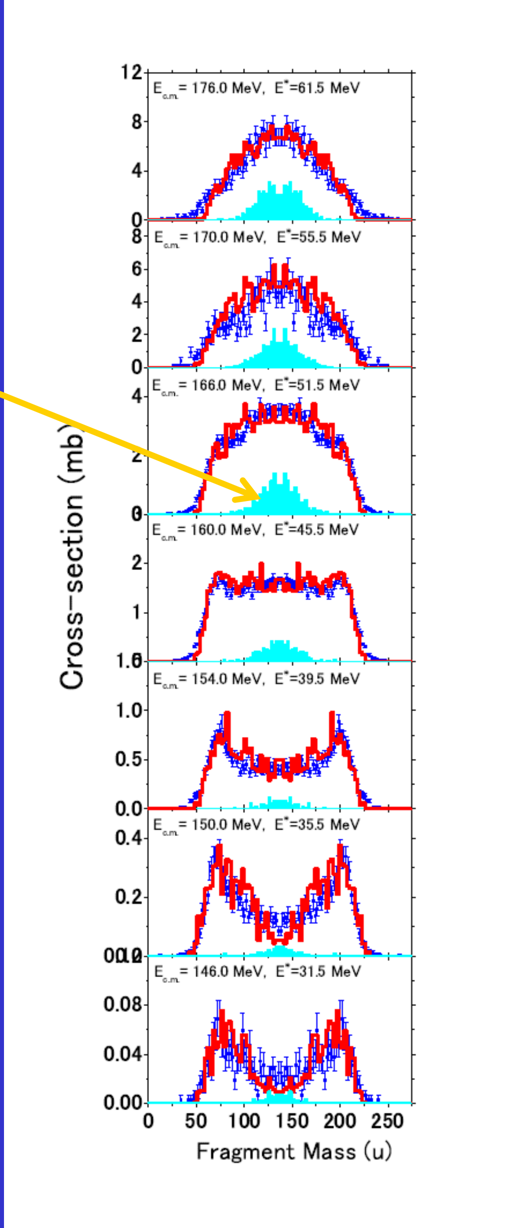
- within $A_{CN}/2 \pm 20$
- Trajectory does not enter the fusion box

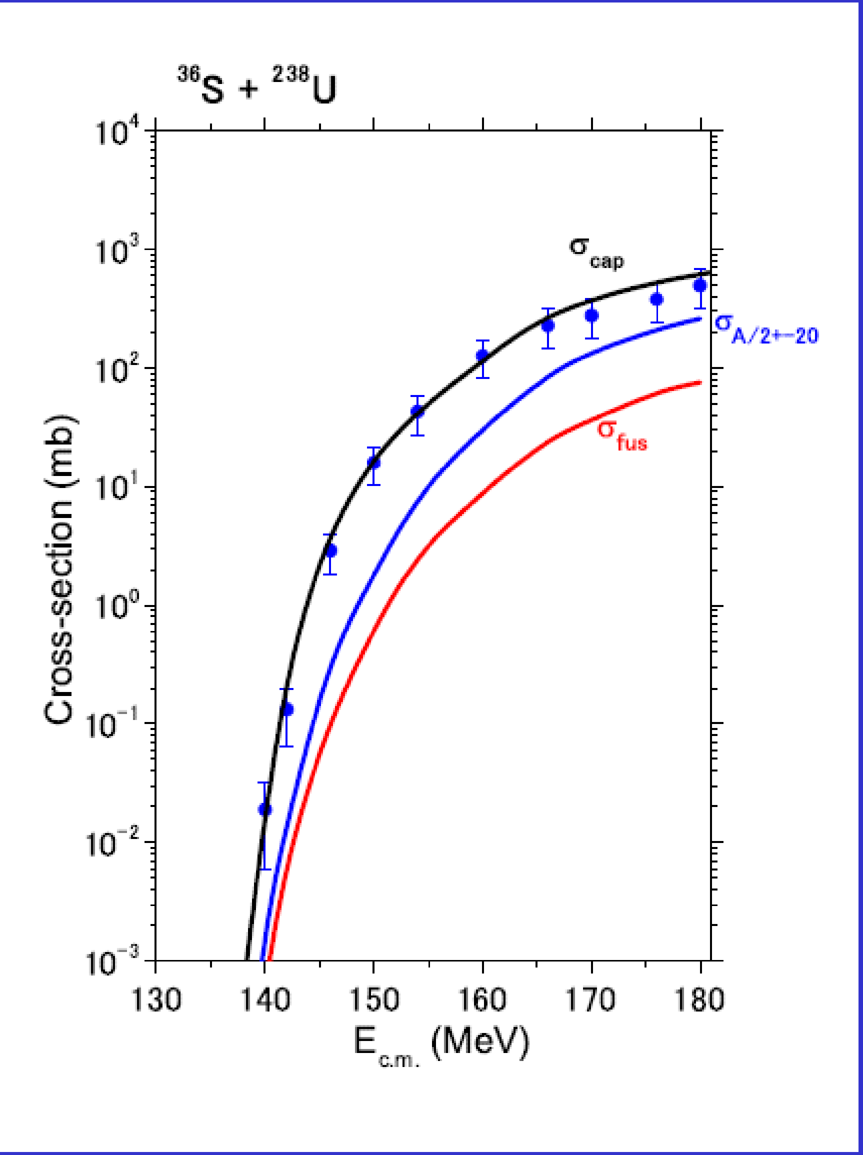
Y. Aritomo and M. Ohta, Nucl. Phys. A 744, 3 (2004)



Results $^{36}S+^{238}U$ MDFF and Cross sections

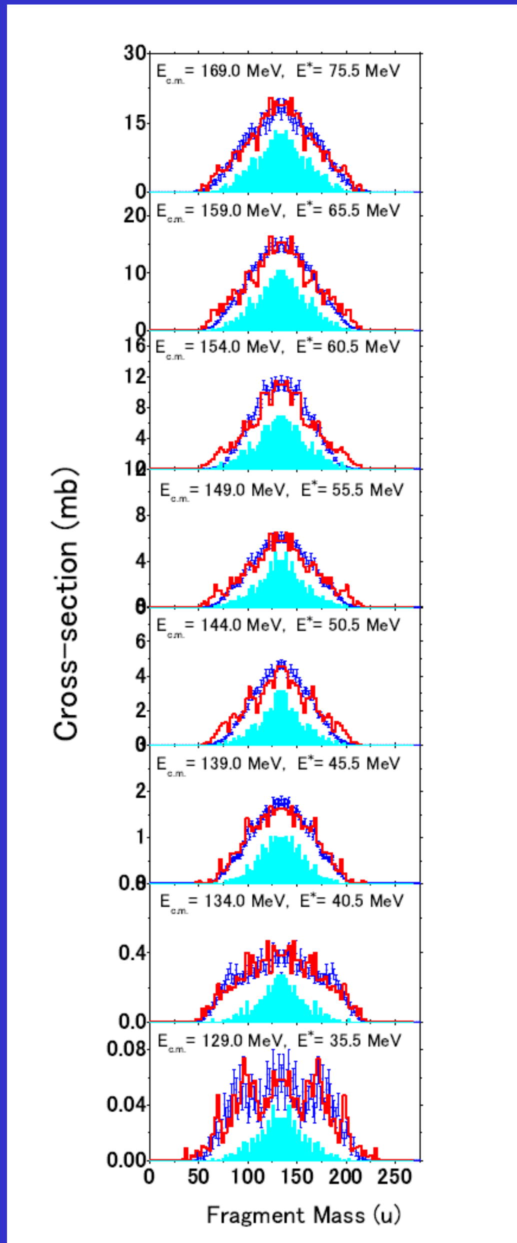
FF process

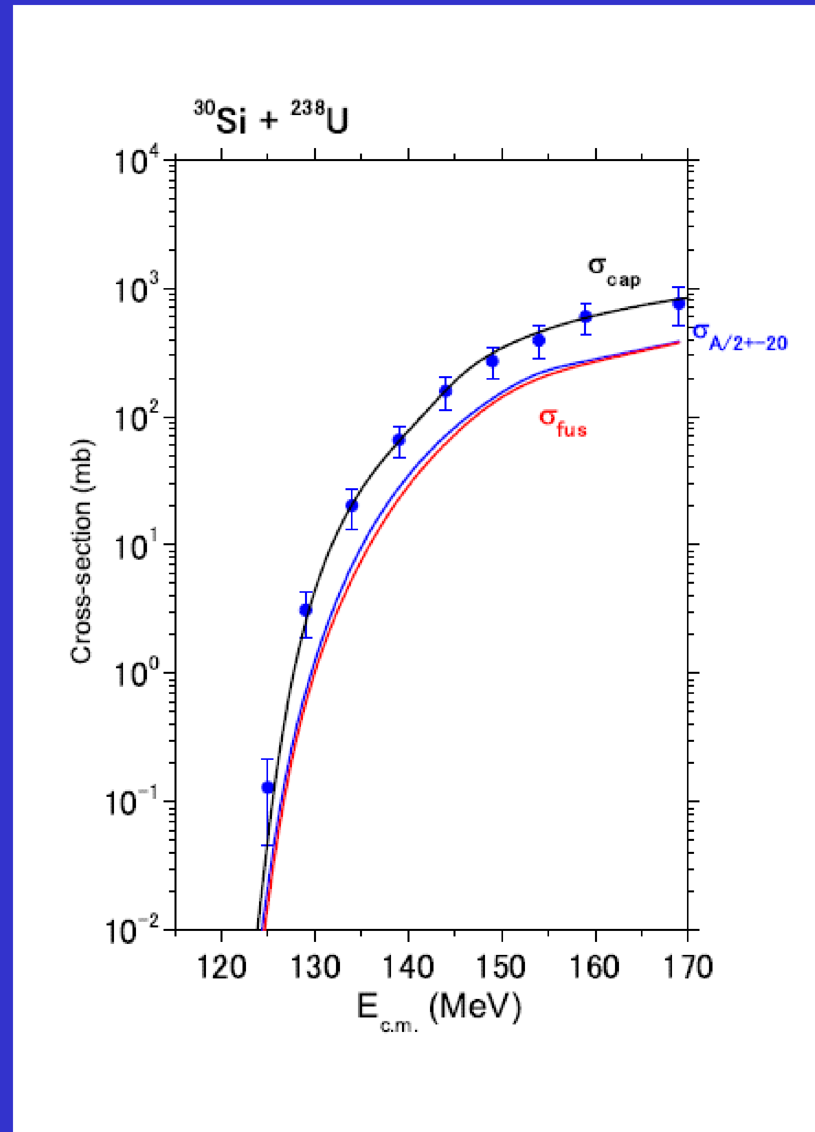




Exp. by
K. Nishio
et al.
at JAEA

Results $^{30}Si+^{238}U$ MDFF and Cross sections



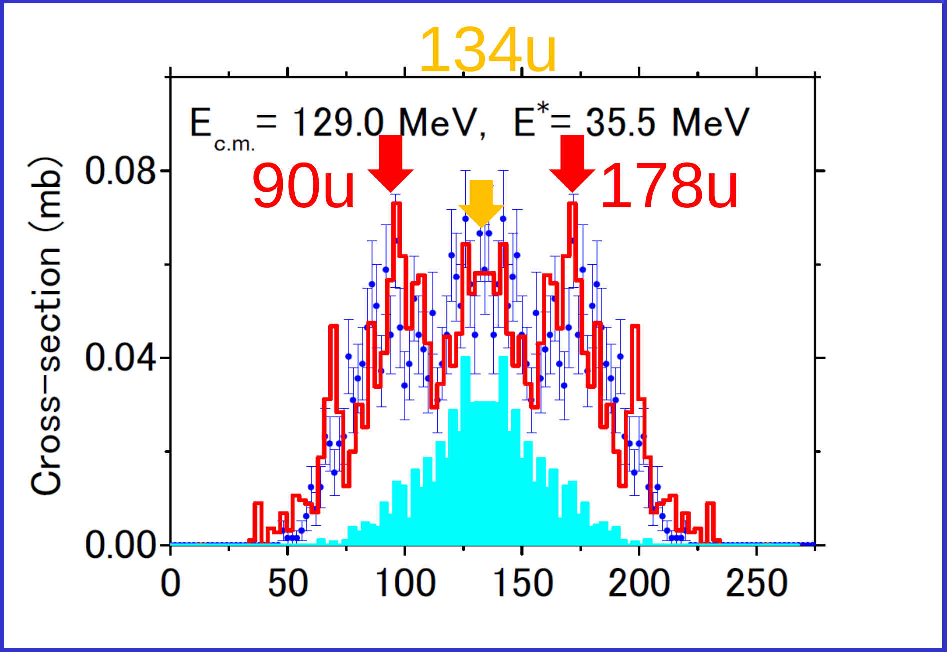


Exp. by
K. Nishio
et al.
at JAEA

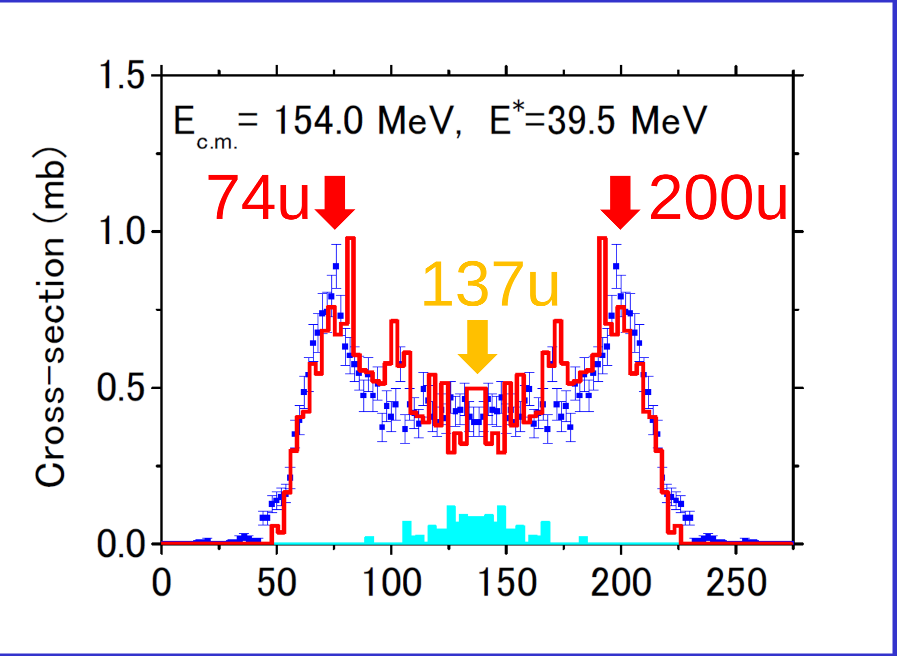
3. Mechanism of Dynamical process

MDFF at Low incident energy

$^{30}Si+^{238}U$

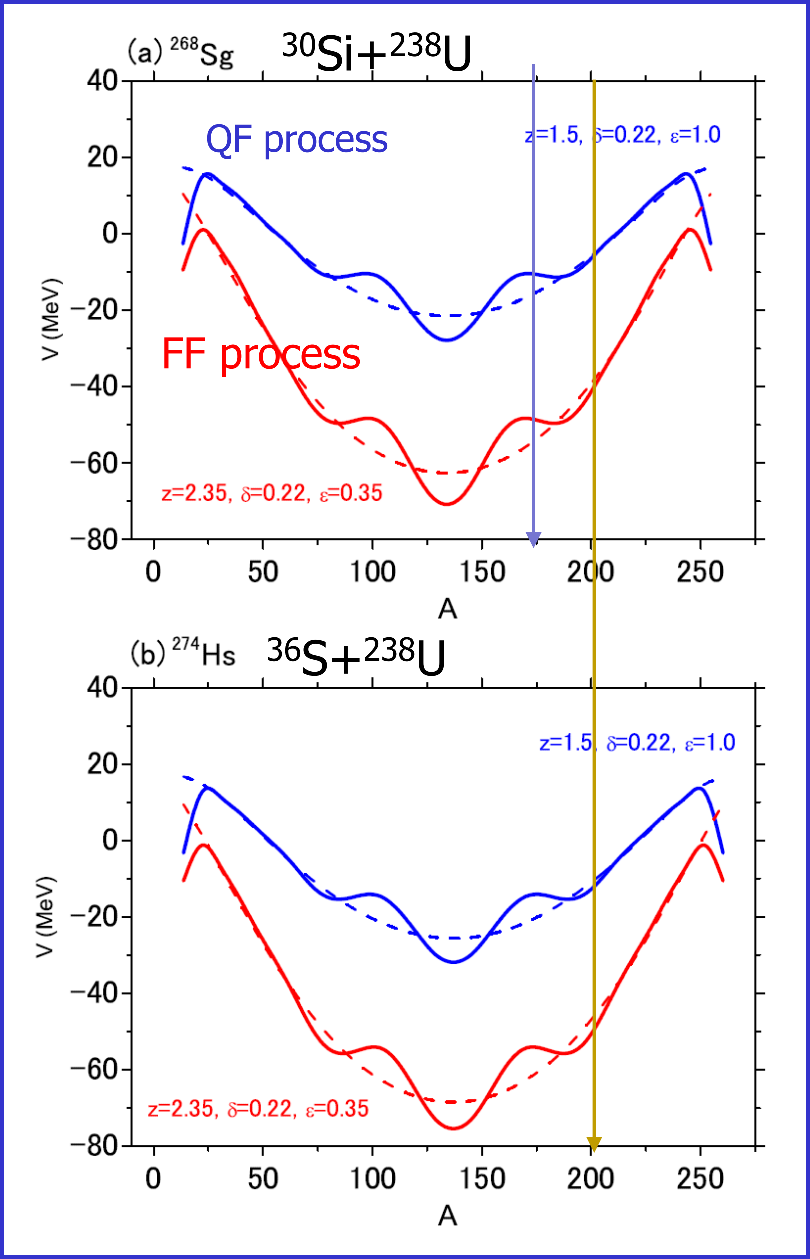


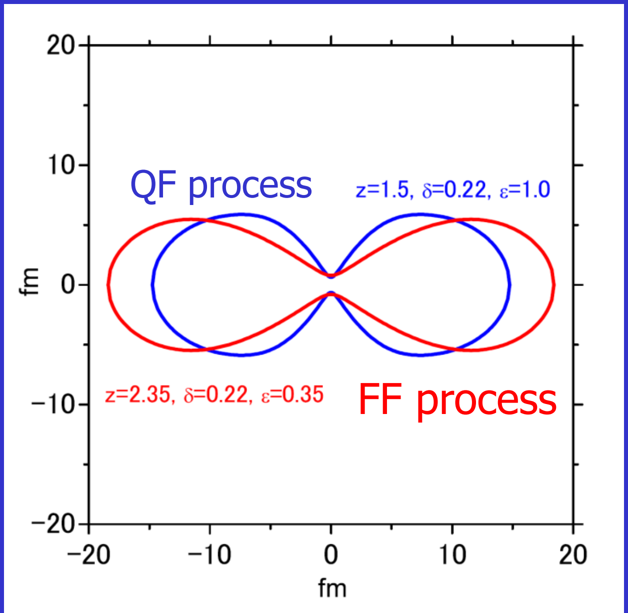
$^{36}S+^{238}U$



Clarification of the mechanism of Fusion-fission process

(a) 1-dim Potential energy on scission line





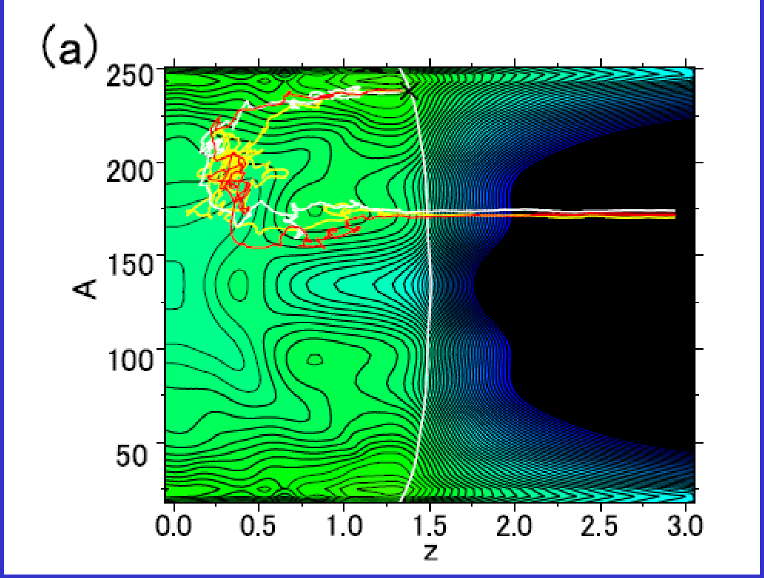
$\alpha = 0$

Nuclear shape at scission point

QF -process

FF-process

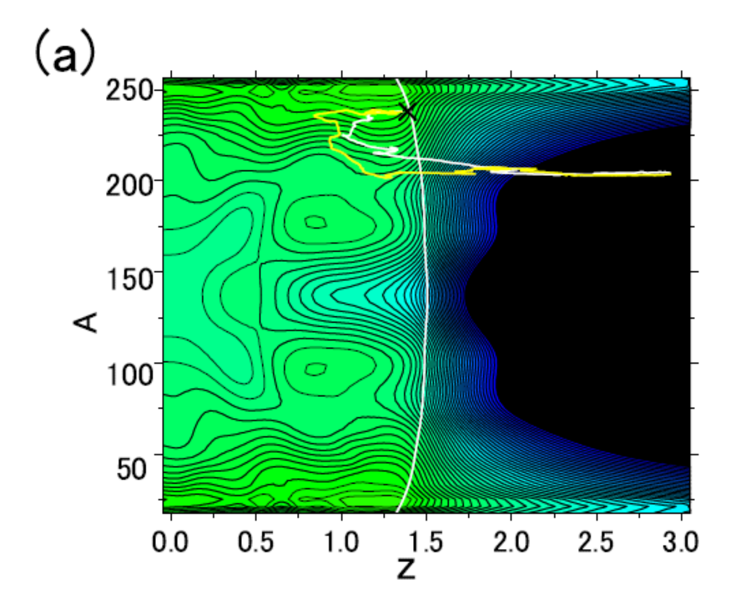
(b) Trajectory Analysis on Potential Energy Surface



Extract the trajectories with A ~ 175 which correspond to the peak of MDFF

$^{30}Si + ^{238}U$

E^* = 35.5 MeV
L=0, θ=0



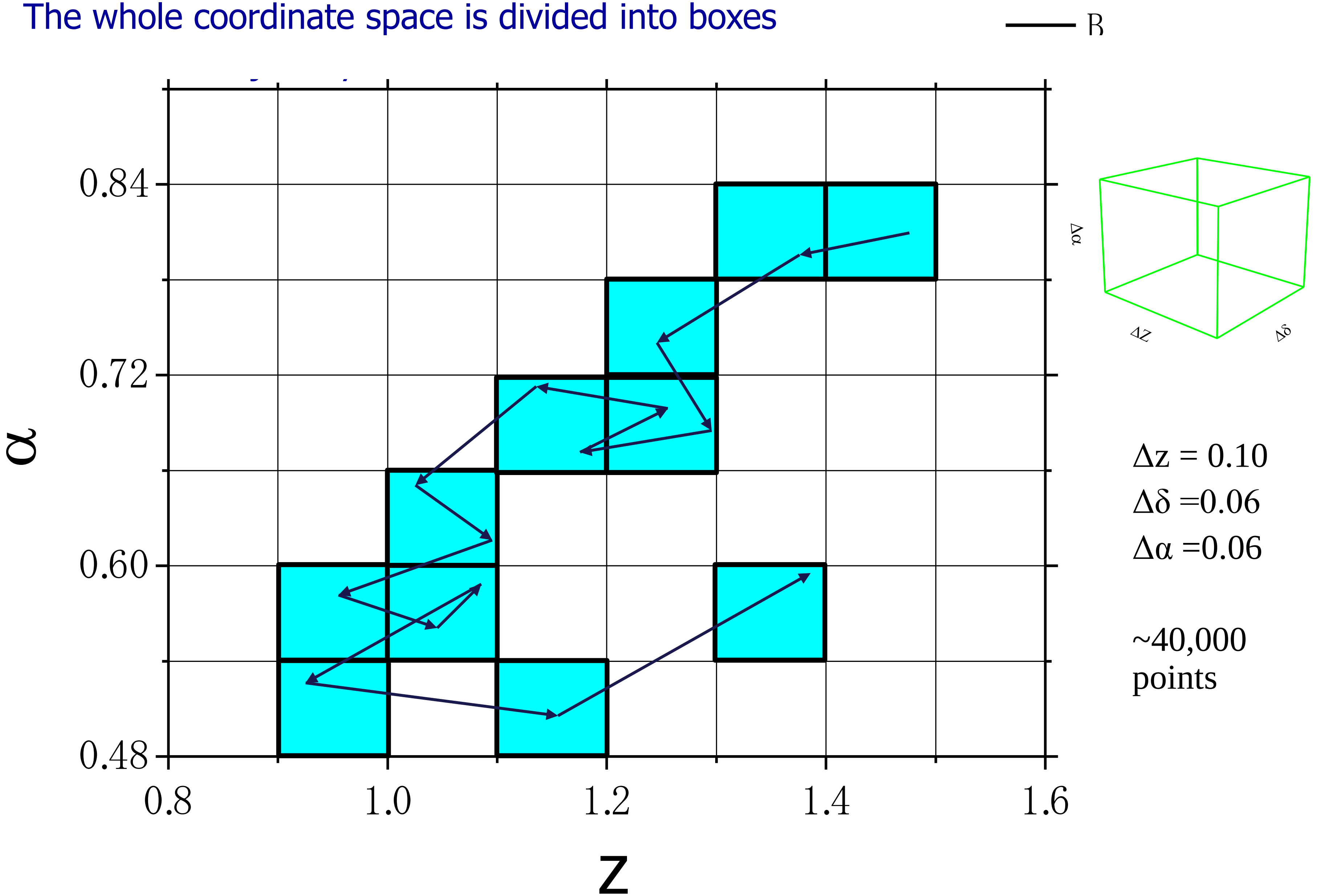
A ~ 200

$^{36}S + ^{238}U$

E^* = 39.5 MeV
L=0, θ=0

Whole Dynamical Process ← using ALL trajectories

The whole coordinate space is divided into boxes



$\Delta z = 0.10$

$\Delta\delta = 0.06$

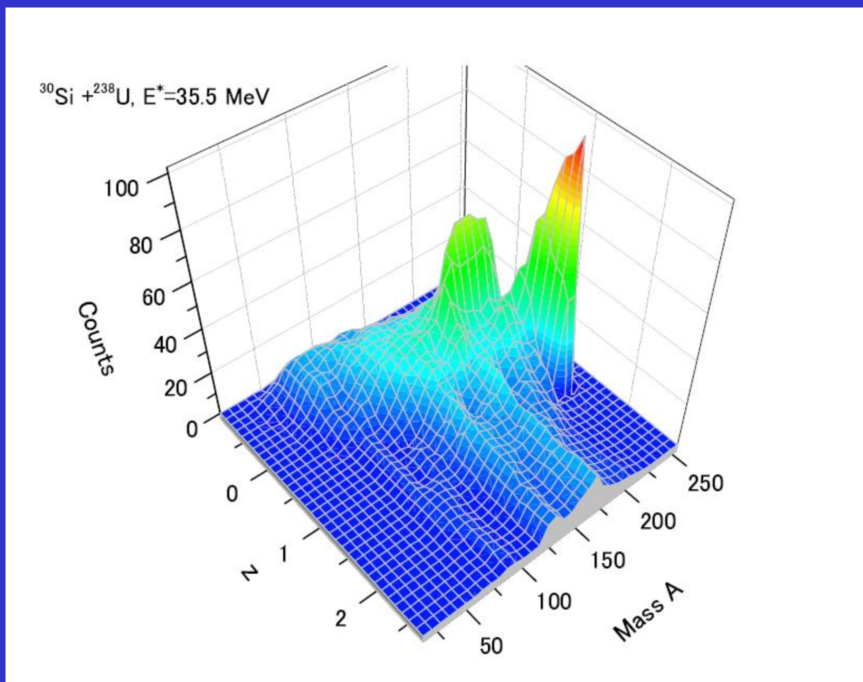
$\Delta\alpha = 0.06$

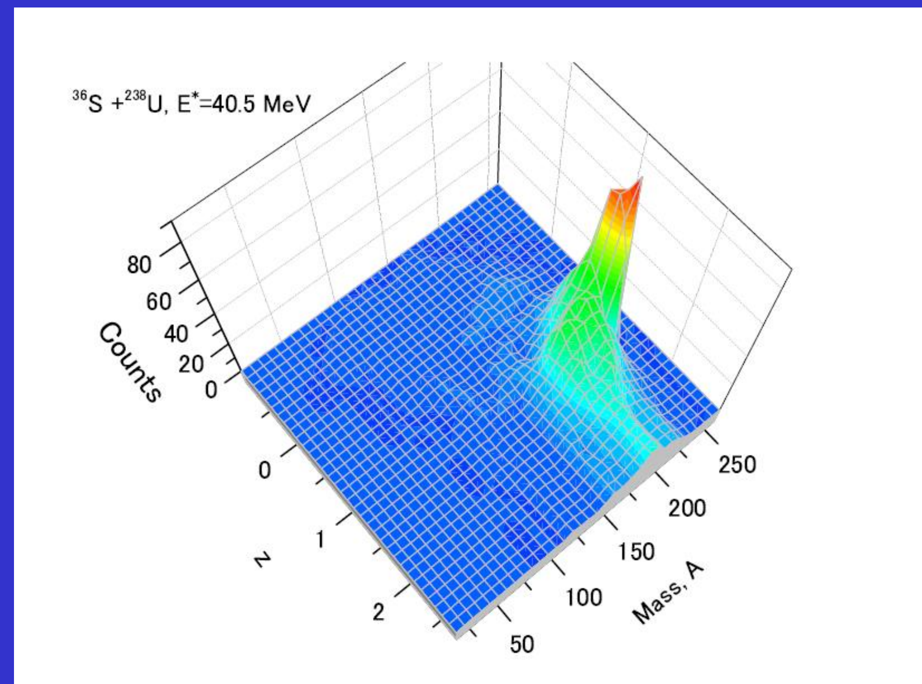
~40,000 points

(c) Trajectory Analysis → "*Probability* Distribution"

$^{30}Si+^{238}U$

$^{36}S+^{238}U$

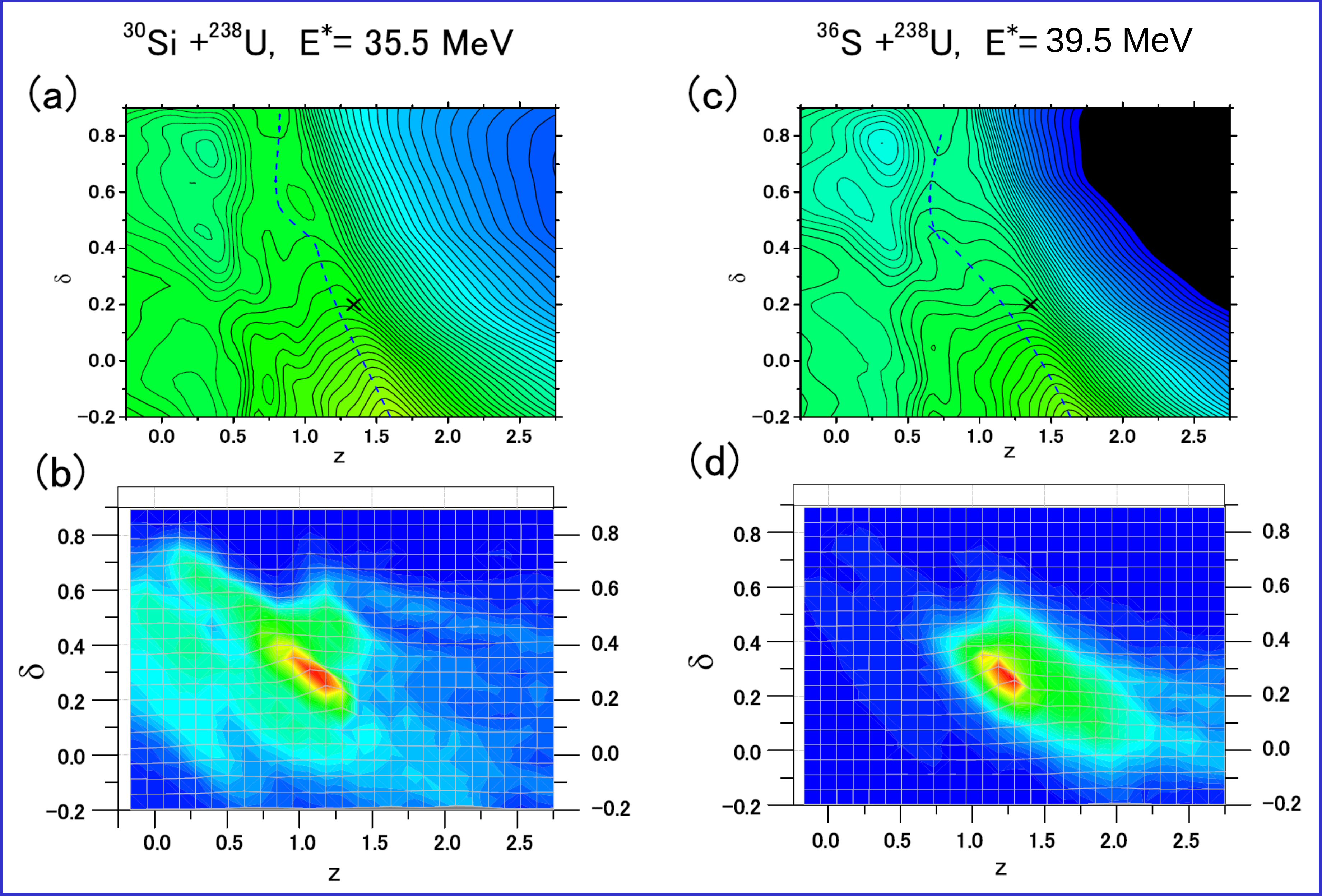




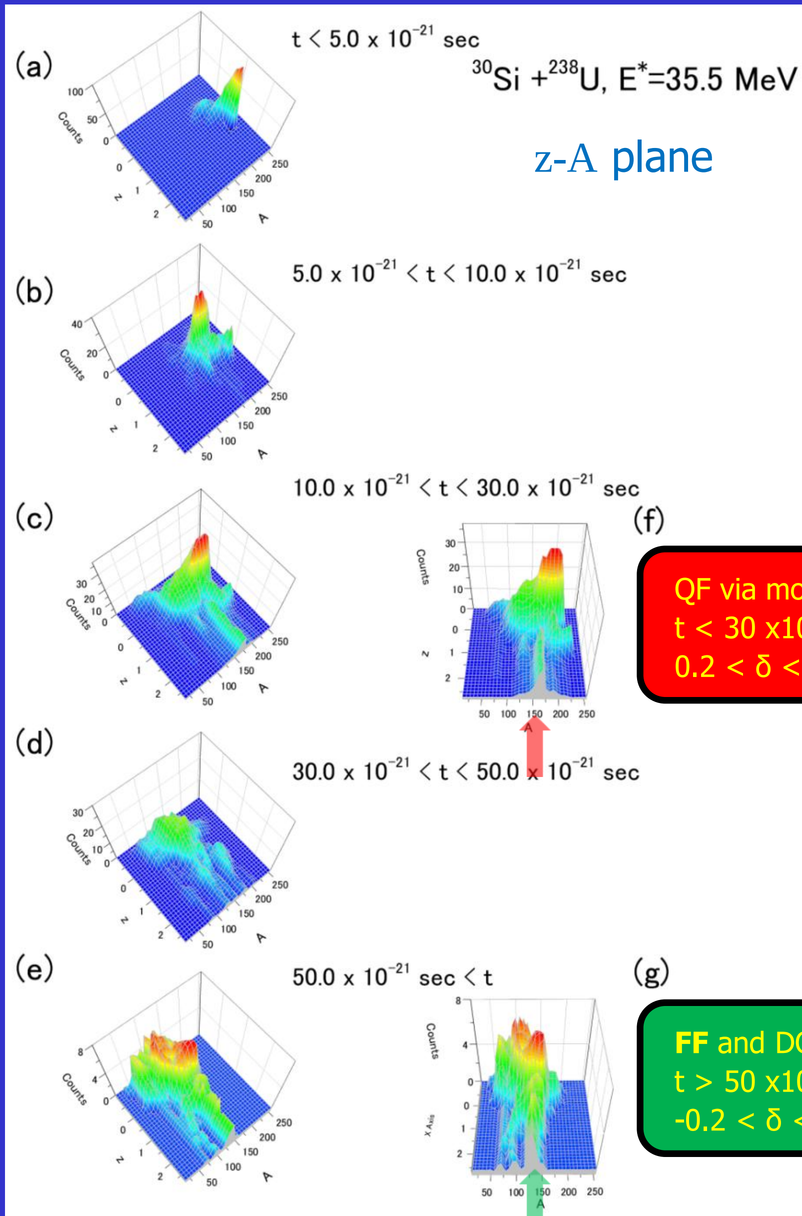
*E** =
35.5 MeV
L=0, θ=0

*E** =
39.5 MeV
L=0, θ=0

Probability distribution on the z-δ plane

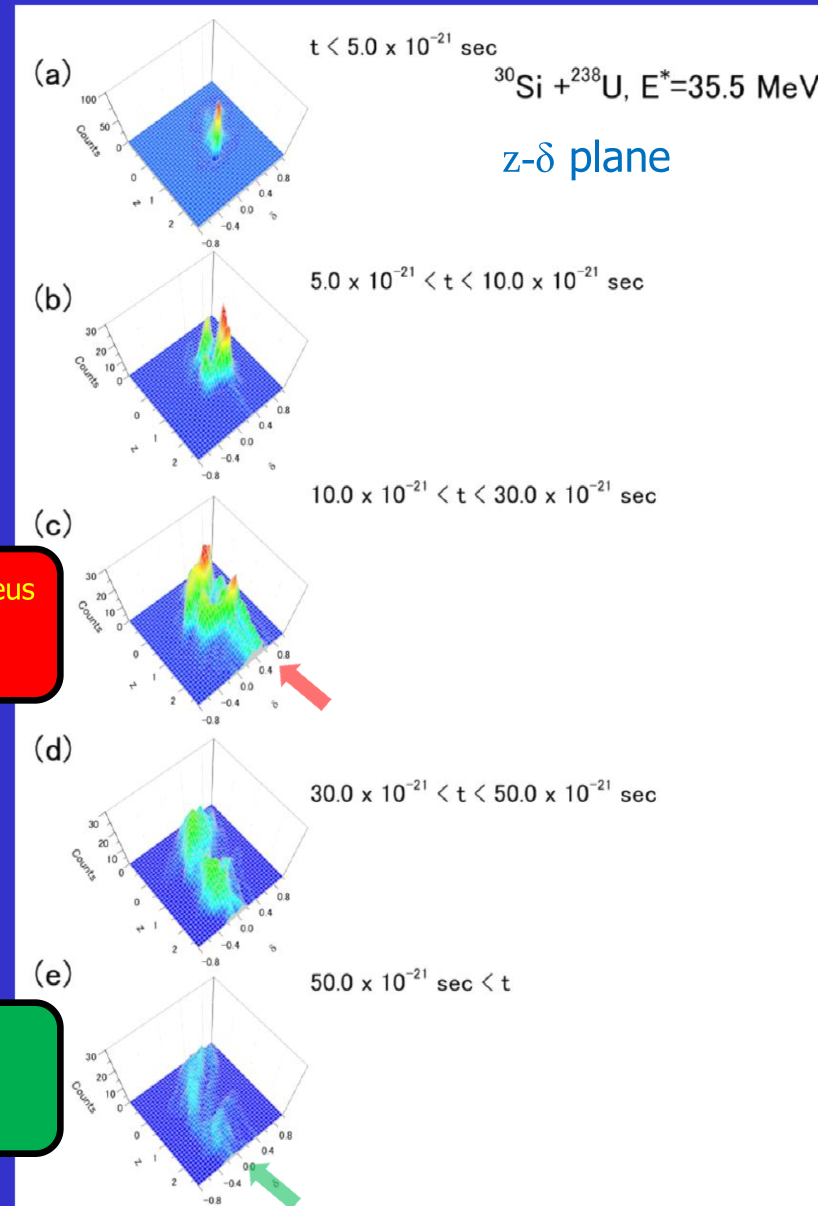


Time evolution of probability distribution

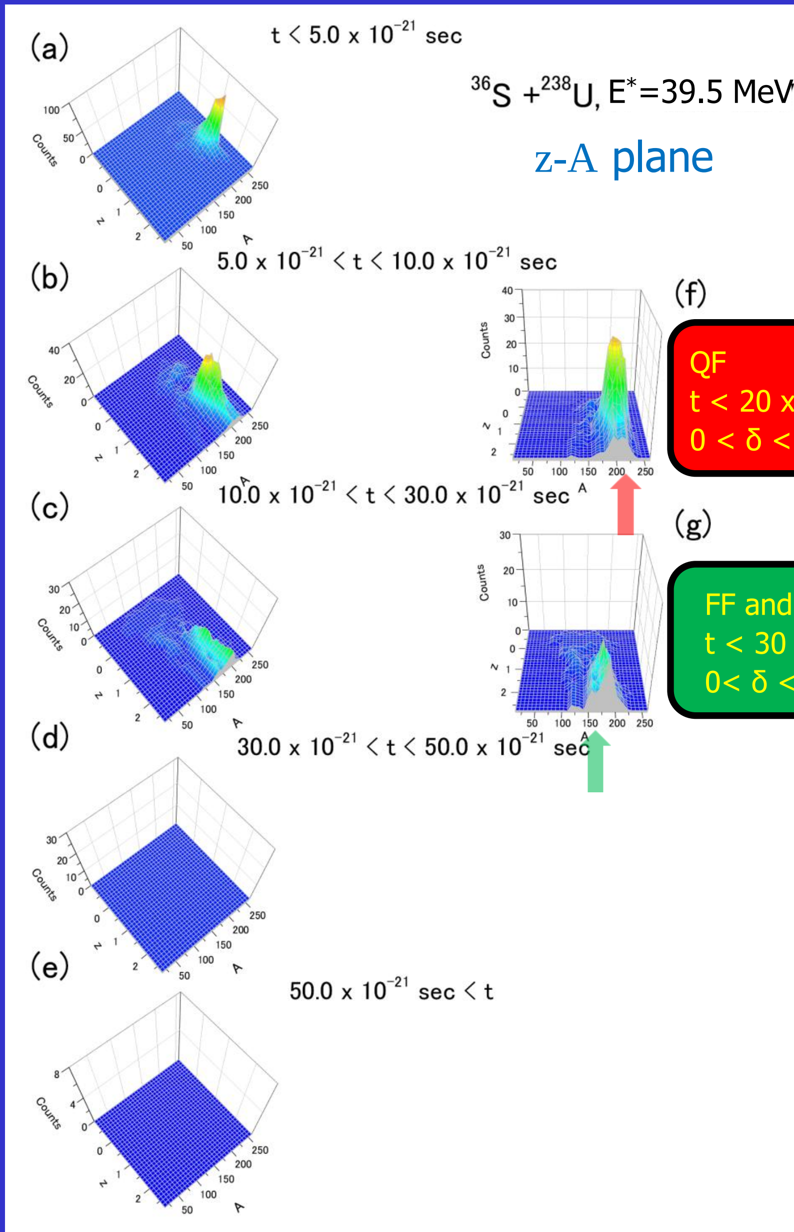


QF via mono-nucleus
t < 30 x10^{-21}
0.2 < δ < 0.5

FF and DQF
t > 50 x10^{-21}
-0.2 < δ < 0.2

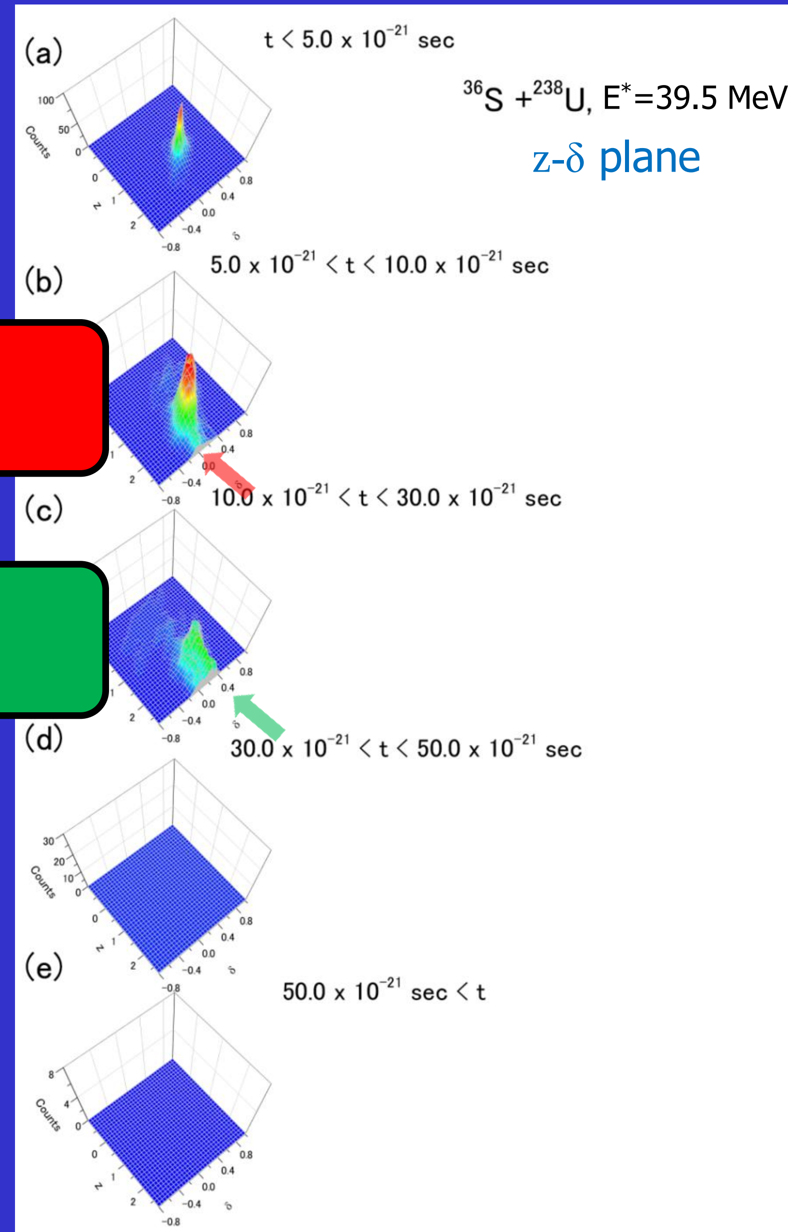


Time evolution of probability distribution

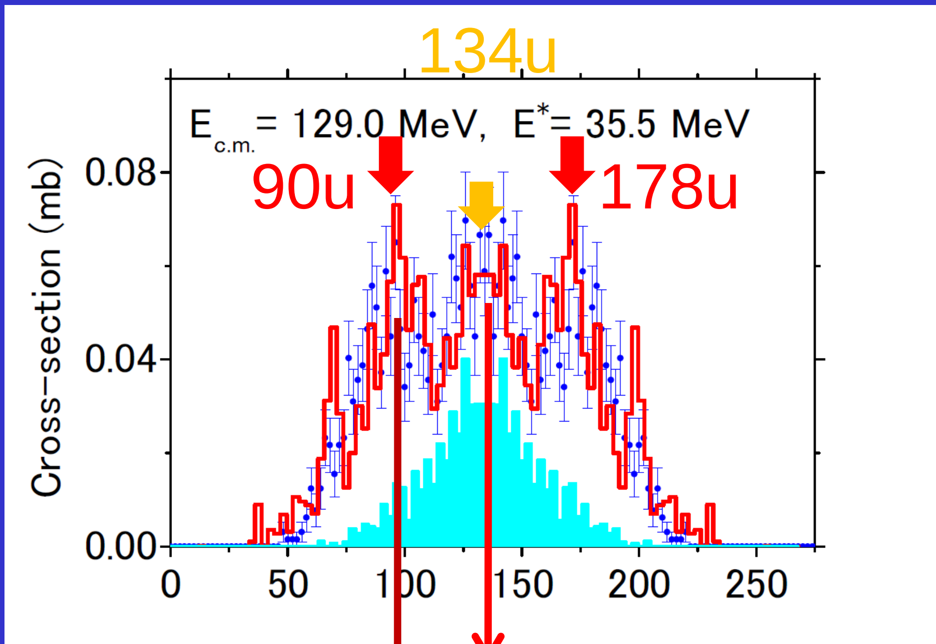


QF
$t < 20 \times 10^{-21}$
$0 < \delta < 0.2$

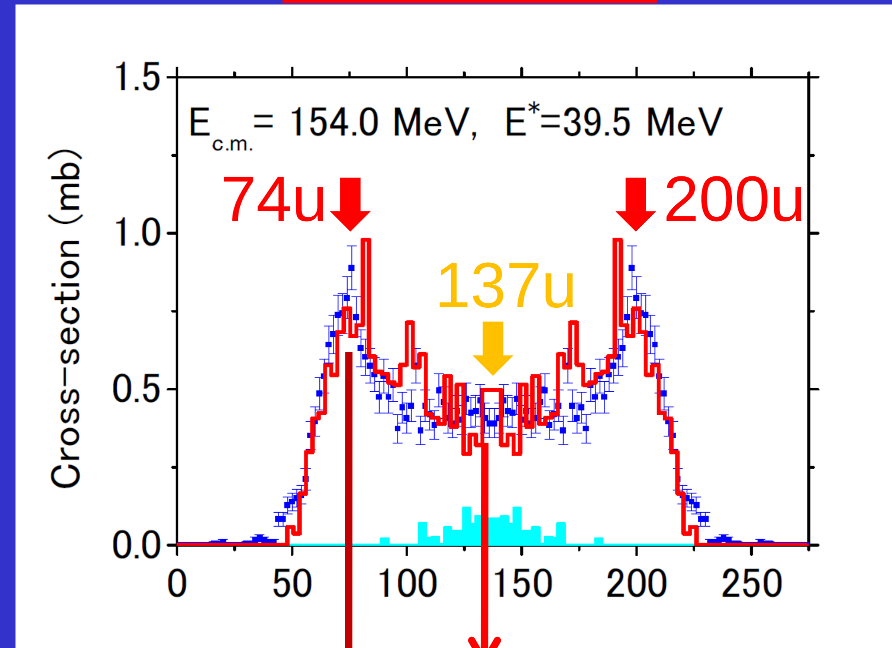
FF and **DQF**
$t < 30 \times 10^{-21}$
$0 < \delta < 0.4$



^{30}Si + ^{238}U



^{36}S + ^{238}U



FF and DQF
t > 50 x 10^{-21} sec
-0.2 < δ < 0.2 (peak 0)

FF and **DQF**
t < 30 x 10^{-21} sec
0 < δ < 0.4 (peak 0.2)

QF via mono-nucleus
t < 30 x 10^{-21} sec
0.2 < δ < 0.5 (peak 0.4)

QF
t < 10 x 10^{-21} sec
0 < δ < 0.2 (peak 0)

4. Summary

1. In order to analyze the fusion-fission process in superheavy mass region, we apply the Couple channels method + Langevin calculation.

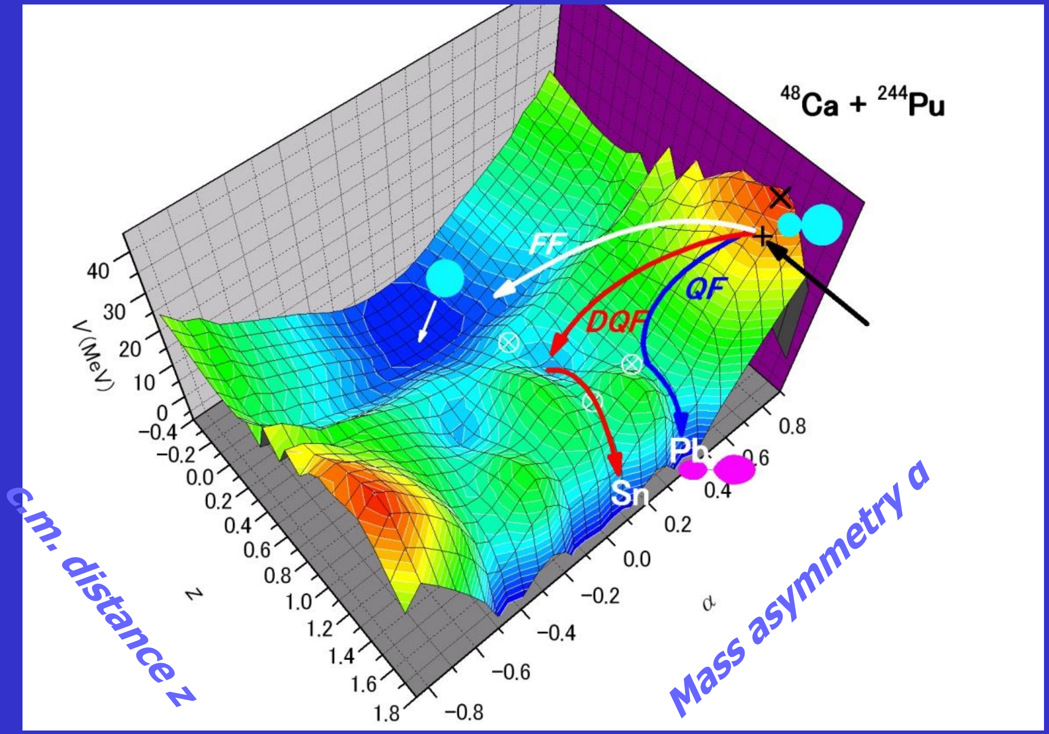
2. Incident energy dependence of mass distribution of fission fragments (MDFF) is reproduced in reaction $^{36}S+^{238}U$ and $^{30}Si+^{238}U$.

3. The shape of the MDFF is analyzed using
   (a) 1-dim potential energy surface on the scission line
   (b) sample trajectory on the potential energy surface
   (c) *probability distribution*

4. The relation between the touching point and the ridge line is very important to decide the process → fusion hindrance

5. Understanding the dynamics of QF and FF processes will be established more realistic model which can predict the opportunity to form wider range of SHE isotope.

Model: Outlook of calculation methods



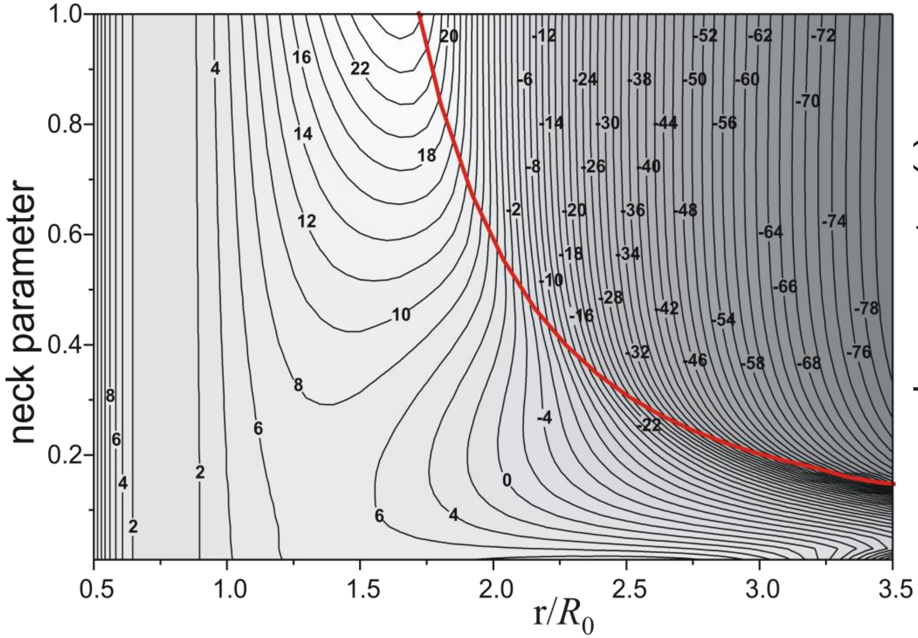
Time-evolution of nuclear shape in fusion-fission process

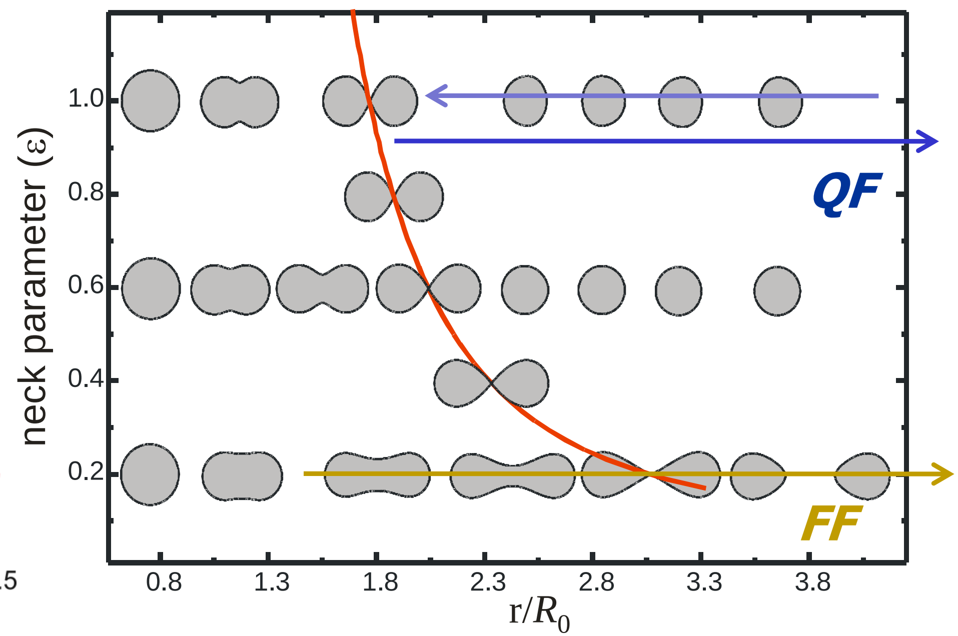
1. Potential energy surface

2. Trajectory → described by equations

Time dependent adiabatic fusion-fission potential

^{224}Th





$$V_{\mathrm{adiab}}(r,\delta,\alpha,\varepsilon;t)=V_{\mathrm{adiab}}(r,\delta,\alpha,\varepsilon=1)\cdot\exp\left(-\frac{t}{\tau_\varepsilon}\right)+V_{\mathrm{adiab}}(r,\delta,\alpha,\varepsilon=\varepsilon_{out})\cdot\left[1-\exp\left(-\frac{t}{\tau_\varepsilon}\right)\right]$$

Time-dependent weight function

$$\tau_\varepsilon = 10^{-20}\ \mathrm{sec}$$

V. Zagrebaev, A. Karpov,
Y. Aritomo, M. Naumenko
and W. Greiner,
Phys. Part. Nucl. 38 (2007) 469

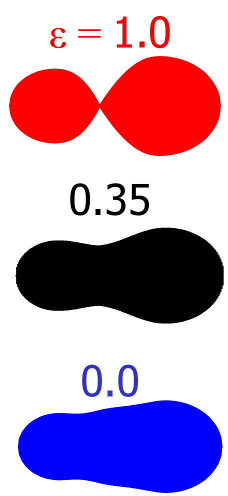
Two Center Shell Model

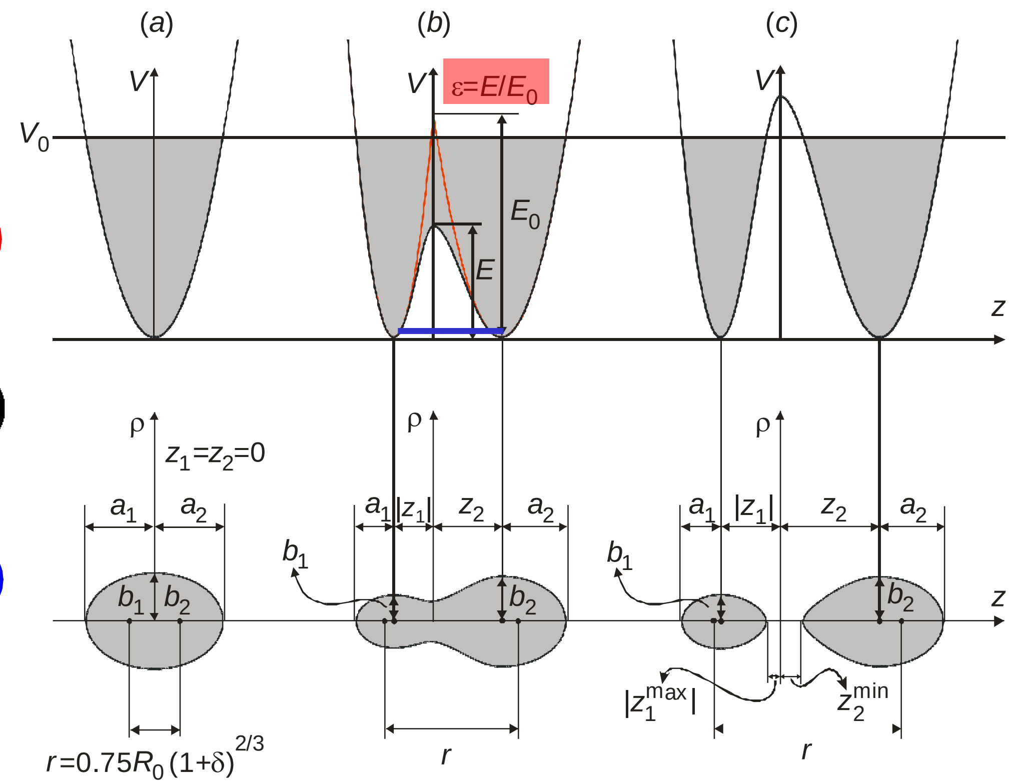
$$\hat{H} = -\frac{\hbar^2}{2m_0}\nabla^2 + V(\mathbf{r}) + V_{\mathrm{LS}}(\mathbf{r},\mathbf{p},\mathbf{s}) + V_{\mathrm{L}^2}(\mathbf{r},\mathbf{p}).$$

$$V(\rho,z) = \frac{1}{2}m_0\begin{cases} \omega_{z1}^2 z'^2 + \omega_{\rho 1}^2\rho^2, \quad z < z_1 \\ \omega_{z1}^2 z'^2\left(1 + c_1 z' + d_1 z'^2\right) + \omega_{\rho 1}^2\left(1 + g_1 z'^2\right)\rho^2, \\ \quad z_1 < z < 0 \\ \omega_{z2}^2 z'^2\left(1 + c_2 z' + d_2 z'^2\right) + \omega_{\rho 2}^2\left(1 + g_2 z'^2\right)\rho^2, \\ \quad 0 < z < z_2 \\ \omega_{z2}^2 z'^2 + \omega_{\rho 2}^2\rho^2, \quad z > z_2, \end{cases}$$

Neck parameter is the ratio of smoothed potential height to the original one where two harmonic oscillator potential cross each other

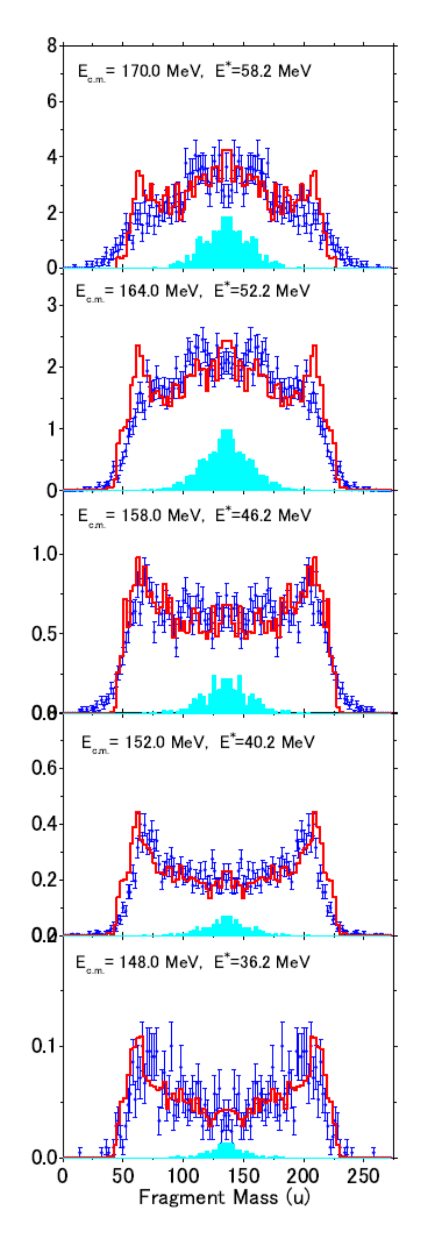
$$z' = \begin{cases} z - z_1 & z < 0 \\ z - z_2 & z > 0 \end{cases}$$

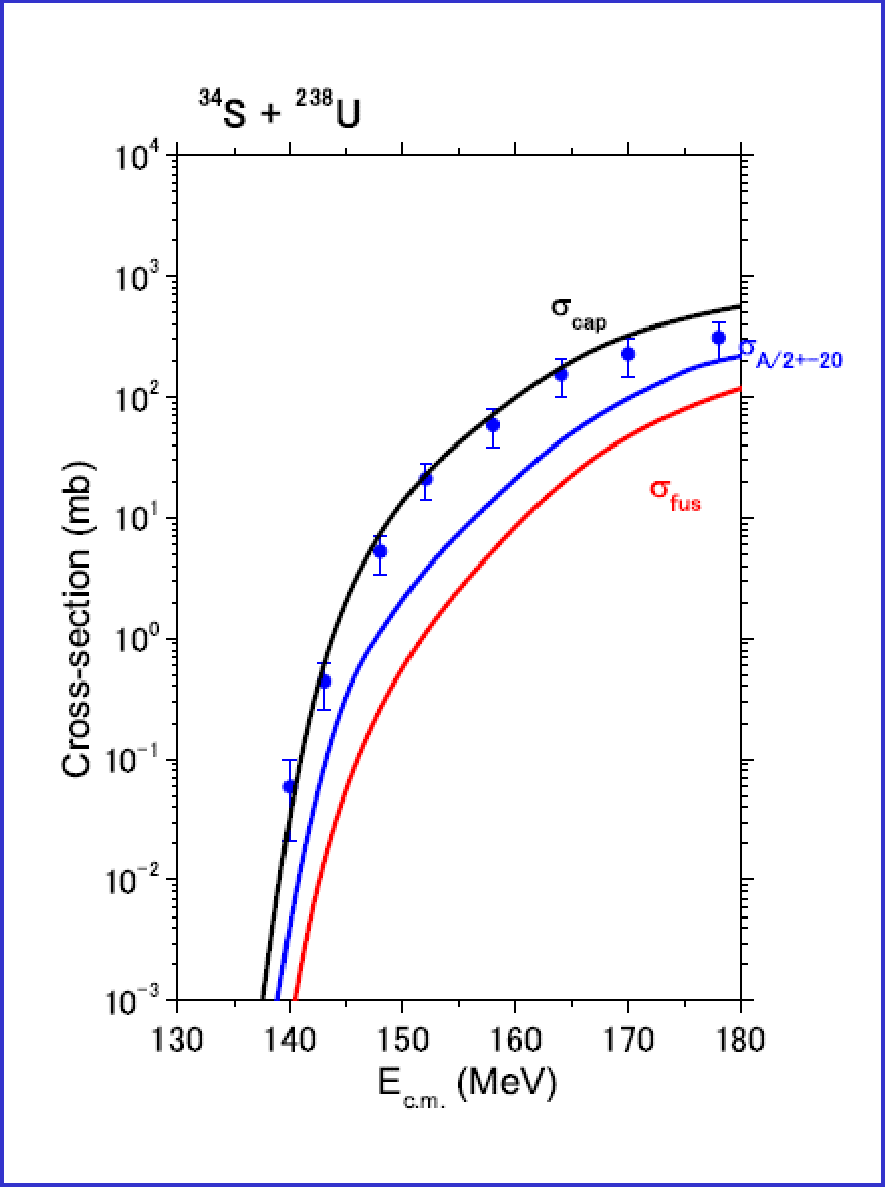




J. Maruhn and W. Greiner, Z. Phys, 1972

Results $^{34}S+^{238}U$ MDFF and Cross sections





What we can obtain under the conditions

Phenomenalism

Dynamical Model based on Fluctuation-dissipation theory

(Langevin eq, Fokker-Plank eq, etc) ← Classical trajectory analysis

We can obtain.... *Fission, Synthesis of SHE*

Mass and TKE distribution of fission fragments *A_{CN} : 200~300*

Neutron multiplicity

Charge distribution

Cross section (capture, mass symmetric fission, fusion)

Angle of ejected particle, Kinetic energy loss (← two body)

Conditions

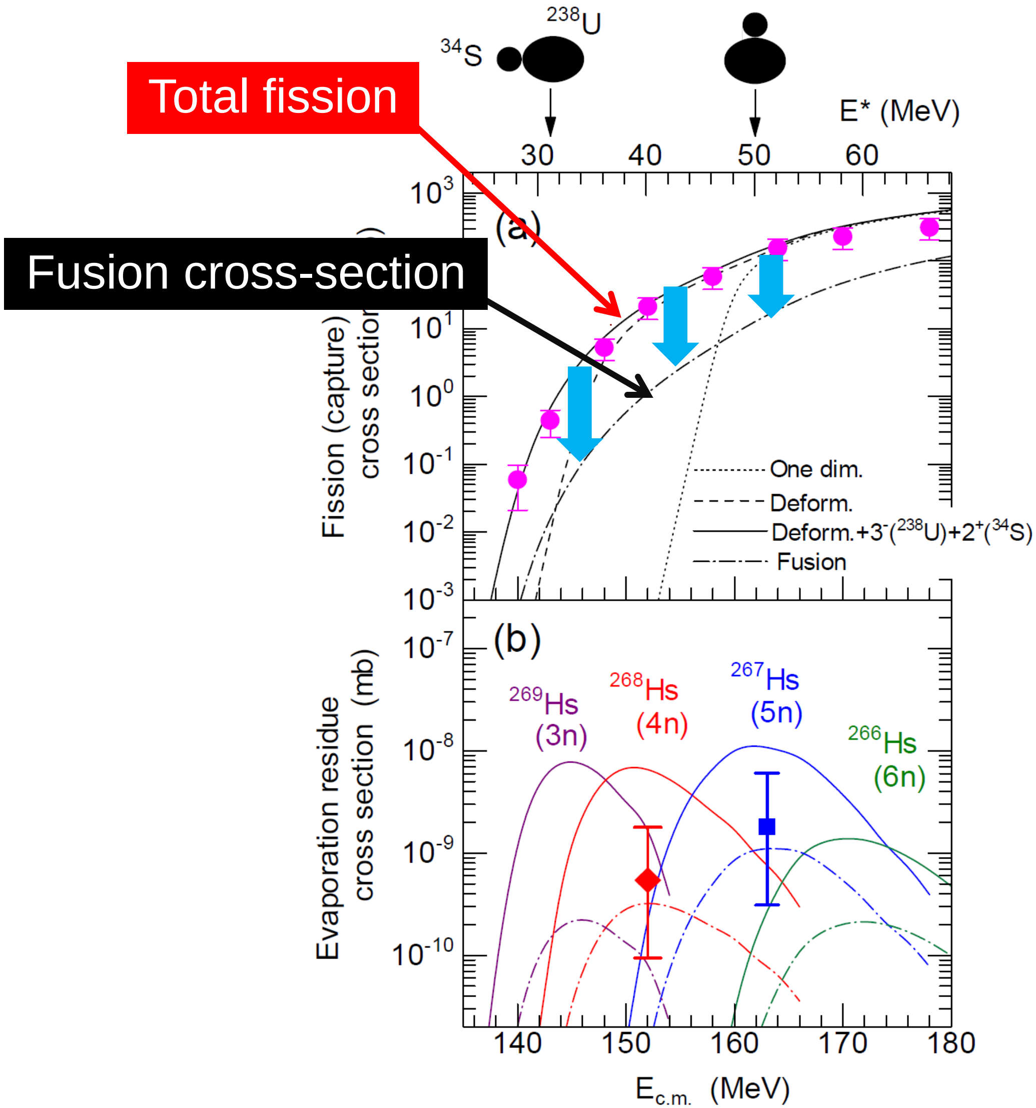
Nuclear shape parameter

Potential energy surface (LDM, shell correction energy, LS force)

Transport coefficients (friction, inertia mass) ← Liner Response Theory

Dynamical equation (memory effect, Einstein relation)

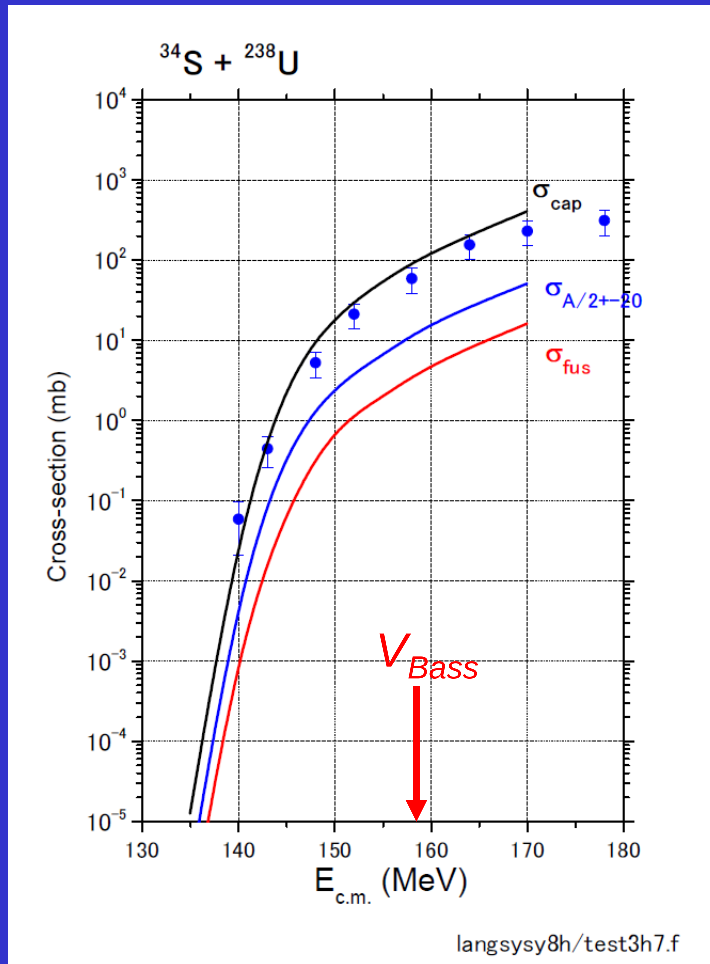
Evaporation residue cross section

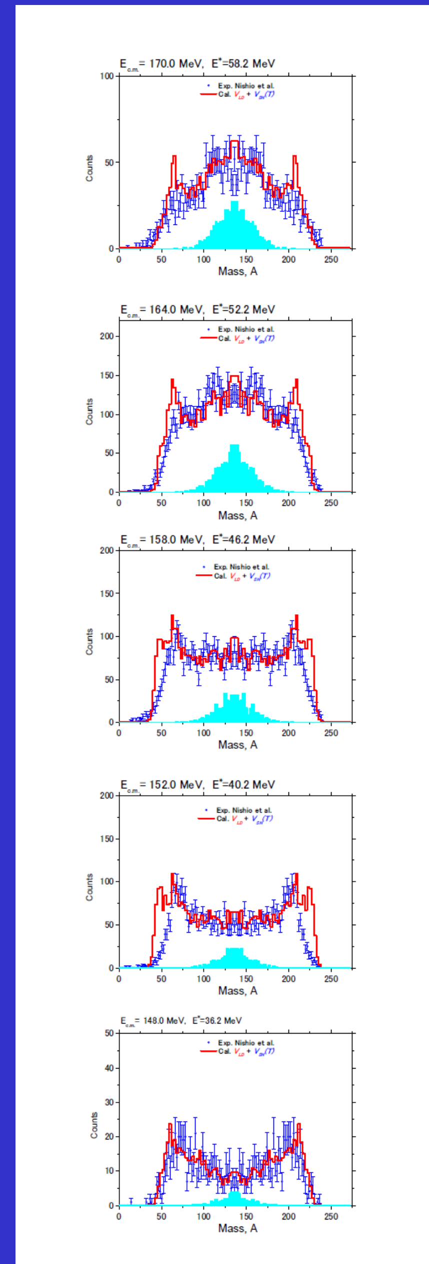


^{34}S + ^{238}U

K.Nishio et al., Phys.Rev.C, 82, 024611 (2010).

Estimation of cross sections under Bass barrier region





Ecm=170
E* =58.2

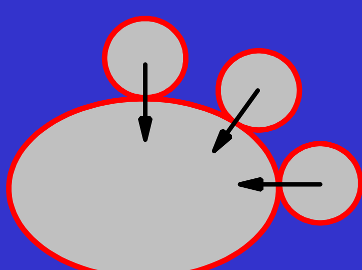
Ecm=164
E* =52.2

Ecm=158
E* =46.2

Ecm=152
E* =40.2

Ecm=148
E* =36.2

Taking into account the contributions of all configurations

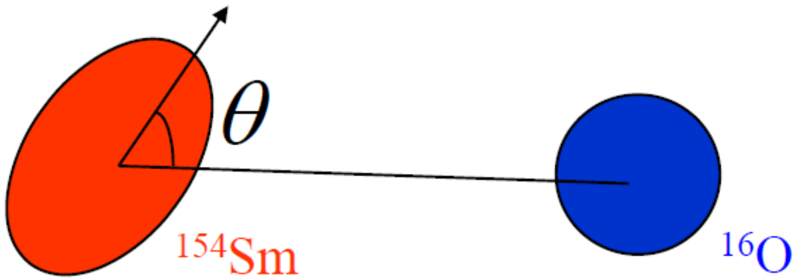


Touching probability
← CC method
+
After touching
← Langevin calculation

Aritomo,
Hagino and Nishio

２つの極限: (ii) 瞬間極限

$$\epsilon \to 0$$



$$\epsilon_I = I(I+1)\hbar^2/2\mathcal{J}$$

$$\mathcal{J} \to \infty$$

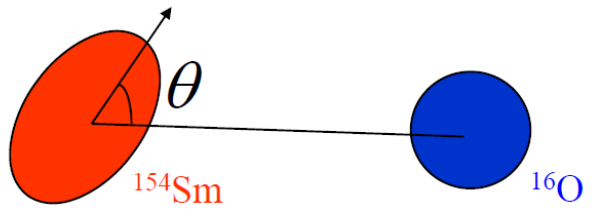
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\text{fus}}(E;\theta)$$

Coupled-channels:

$$\begin{pmatrix} 0 & f(r) & 0 \\ f(r) & \frac{2\sqrt{5}}{7} f(r) & \frac{6}{7} f(r) \\ 0 & \frac{6}{7} f(r) & \frac{20\sqrt{5}}{77} f(r) \end{pmatrix} \xrightarrow{\text{diagonalize}} \begin{pmatrix} \lambda_1(r) & 0 & 0 \\ 0 & \lambda_2(r) & 0 \\ 0 & 0 & \lambda_3(r) \end{pmatrix}$$

$$\Rightarrow \quad P(E) = \sum_i w_i\, P(E; V_0(r) + \lambda_i(r))$$

Slow intrinsic motion
→ Barrier Distribution



$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

M.A.Nagarajan, A.B. Balantekin, N. Takigawa, PRC 34, 894 (1986)

Coupled-channels:

$$\begin{pmatrix} 0 & f(r) & 0 \\ f(r) & \frac{2\sqrt{5}}{7}f(r) & \frac{6}{7}f(r) \\ 0 & \frac{6}{7}f(r) & \frac{20\sqrt{5}}{77}f(r) \end{pmatrix} \xrightarrow{\text{diagonalize}} \begin{pmatrix} \lambda_1(r) & 0 & 0 \\ 0 & \lambda_2(r) & 0 \\ 0 & 0 & \lambda_3(r) \end{pmatrix}$$

$$\hat{P}_2(\cos\theta)f_\alpha(\theta)=\lambda_\alpha f_\alpha(\theta) \ ,$$

$$\sum_{I'}\frac{[(2I+1)(2I'+1)]^{1/2}}{5}\langle I0I'0|20\rangle^2\beta_{\alpha I'}=\lambda_\alpha\beta_{\alpha I} \ .$$

$$f_\alpha(\theta)=\sum_{I=0}^{I_{\max}}\beta_{\alpha I}Y_{I0}(\theta,\phi) \ ,$$

$f_\alpha(\theta)$ eigenstate of the operator $\hat{P}_2(\cos\theta)$

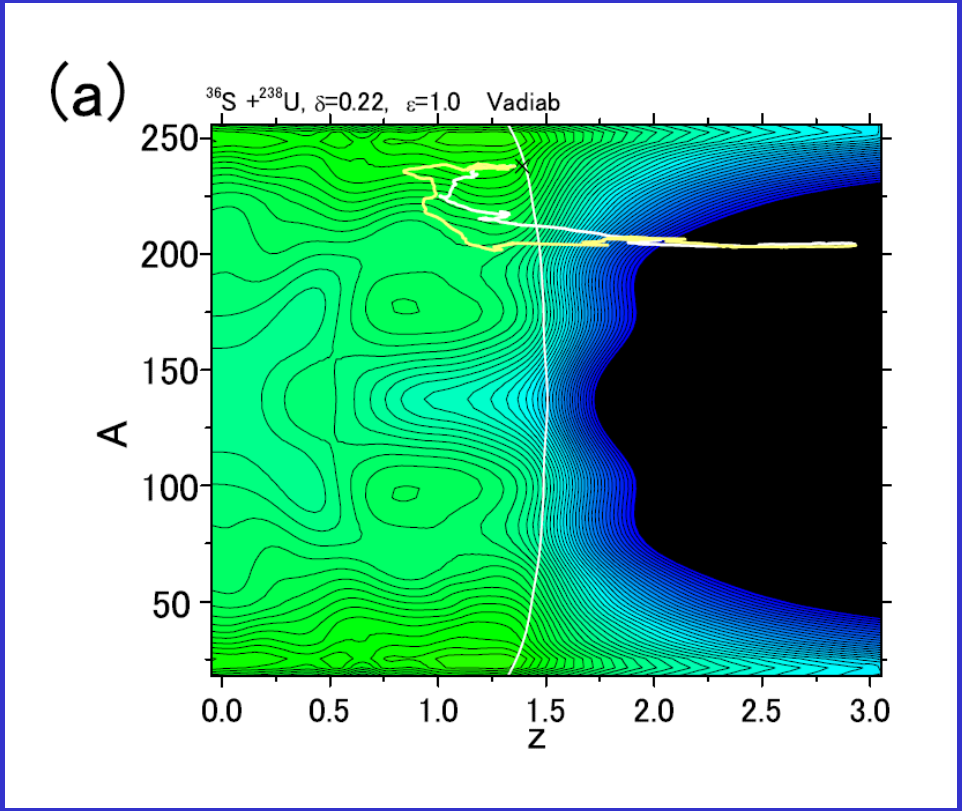
λ_α is the corresponding eigenvalue

$$\sigma_{\rm fusion}^{\rm total}=\int_0^{\pi/2}\sin\theta\,\sigma_{\rm fusion}(\theta)d\theta$$
$$=\sum w_\alpha\sigma_{\rm fusion}(\alpha) \ ,$$

where the abscissas, $\cos\theta_\alpha$, and weight factors, w_α, correspond to Gaussian integration.

Trajectory Analysis on Potential Energy Surface $^{36}S+^{238}U$

*E** =
39.5 MeV
L=0, θ=0



Key Words

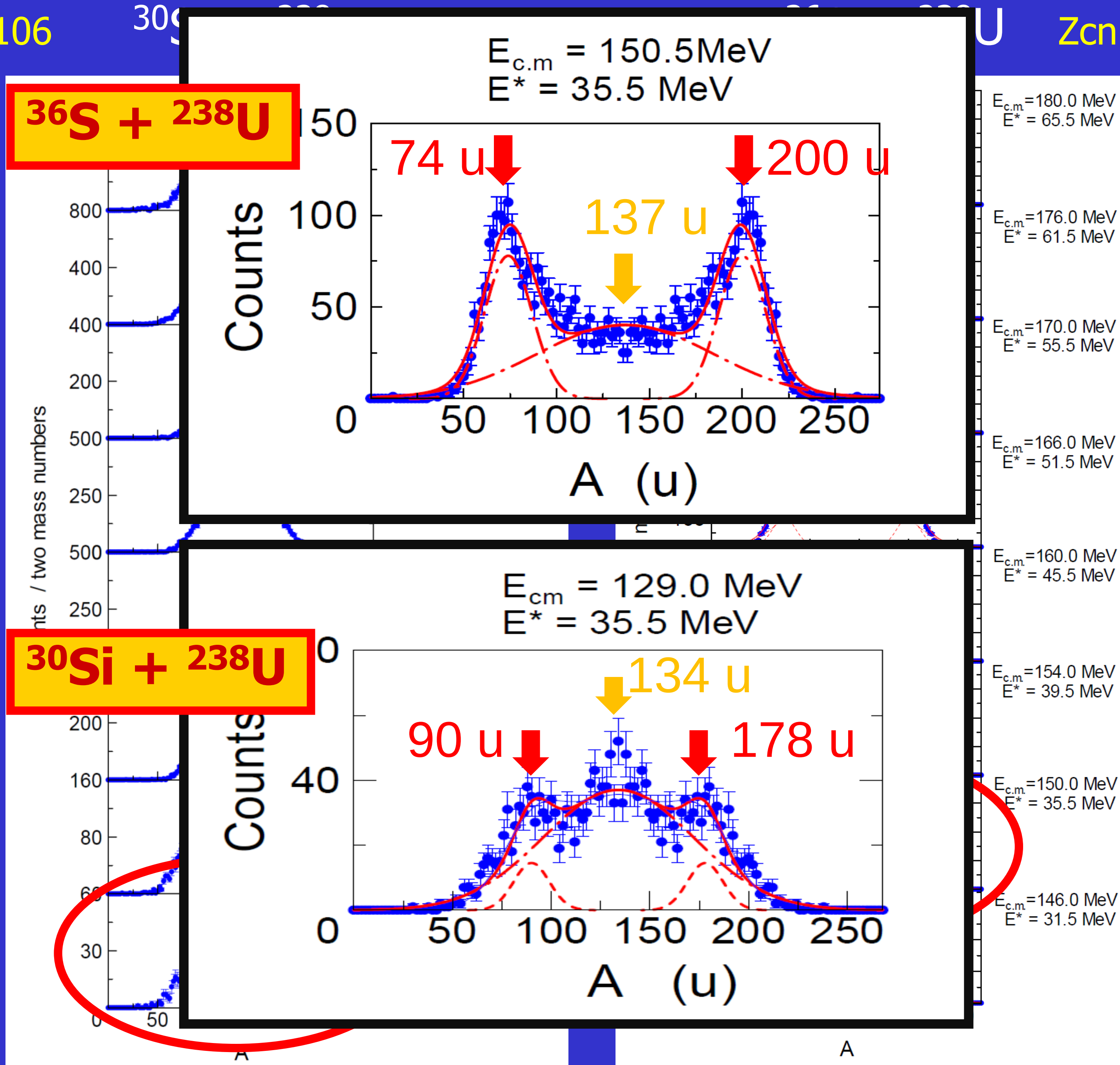
Phenomenalism (現象論)

現象論的理論というのは、

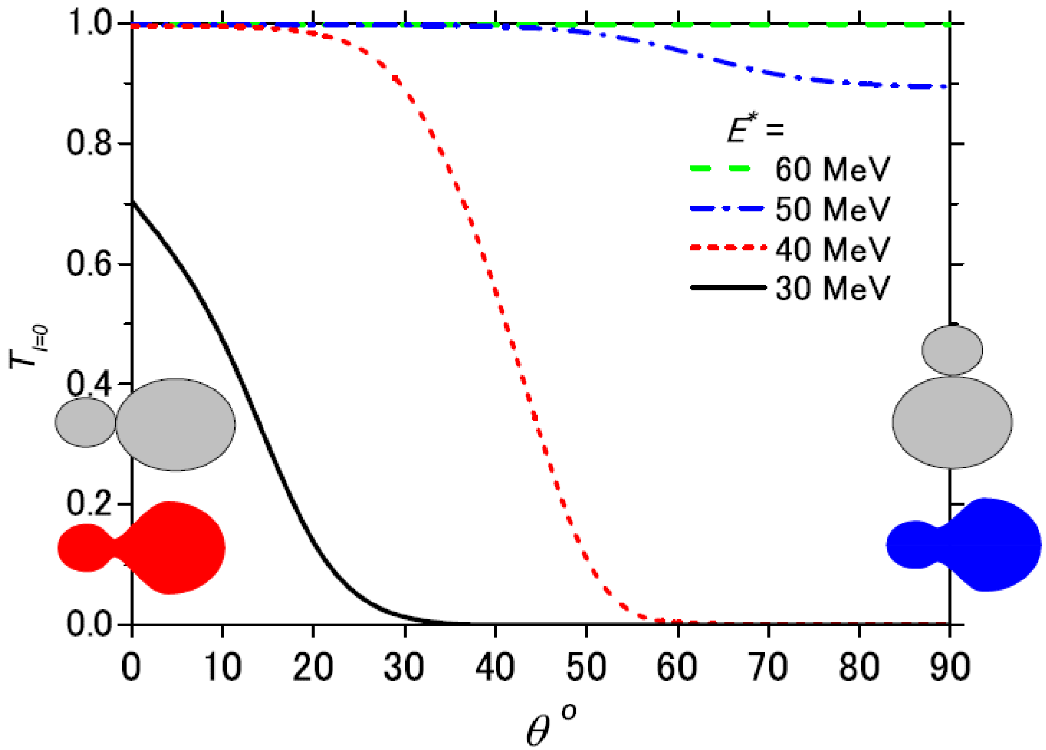
　現象を説明するためにある仮定のもとに理論を展開するにあたり、この仮定が必ずしも厳密に**実在認識**に相当しなくとも、これによって導き出された結果が現象における諸関係をよく説明し得るならば、それで一応満足するが如き理論を言う

竹山説三　著　「電磁気学現象論」

← Zcn=106 Zcn=108 →

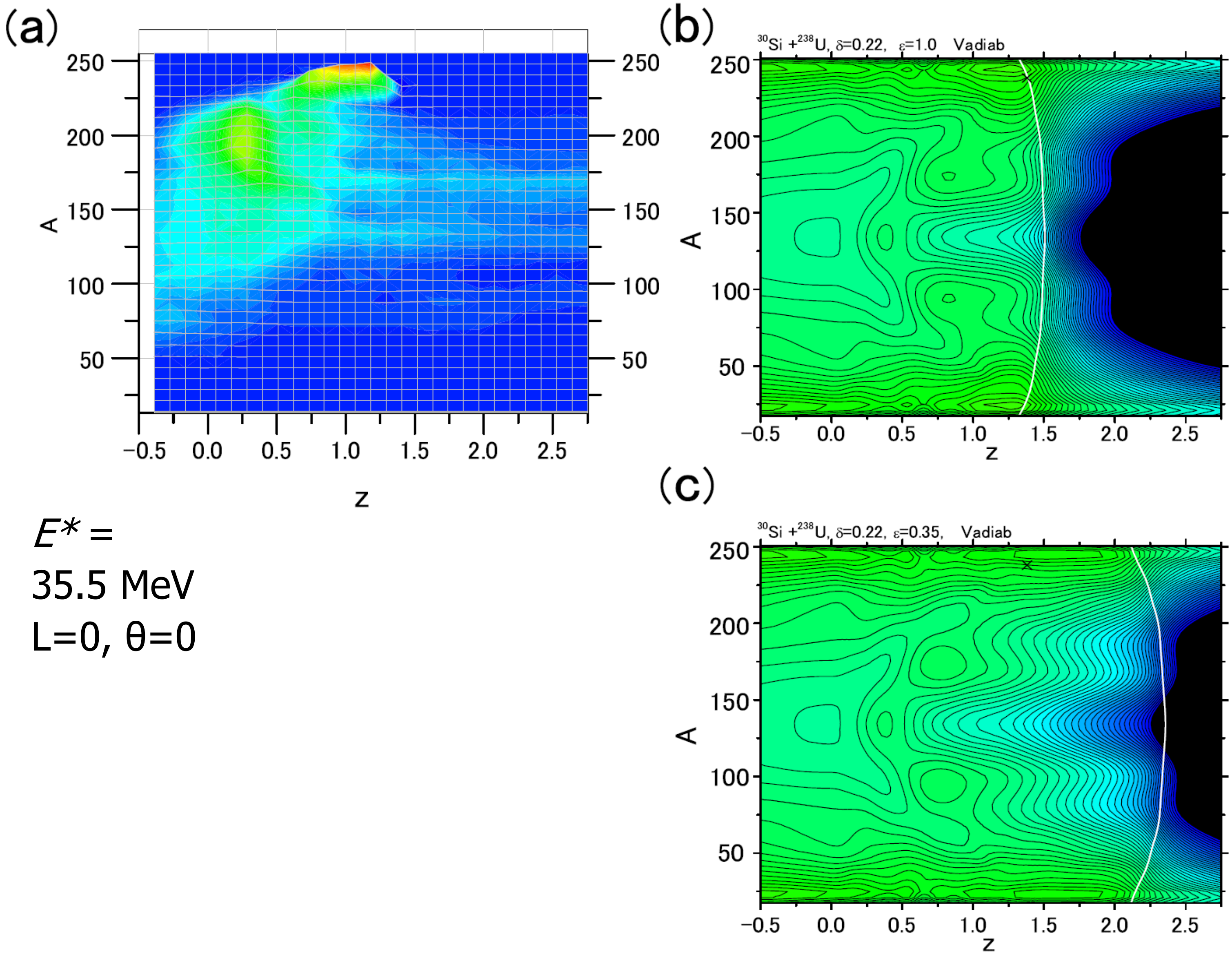


Exp. by
K. Nishio
et al.



1.0
0.8
0.6
0.4
0.2
0.0
$T_{l=0}$
0
10
20
30
40
50
60
70
80
90
θ^o
E^* =
60 MeV
50 MeV
40 MeV
30 MeV

Probability distribution $^{30}Si+^{238}U$ on the z-A plane

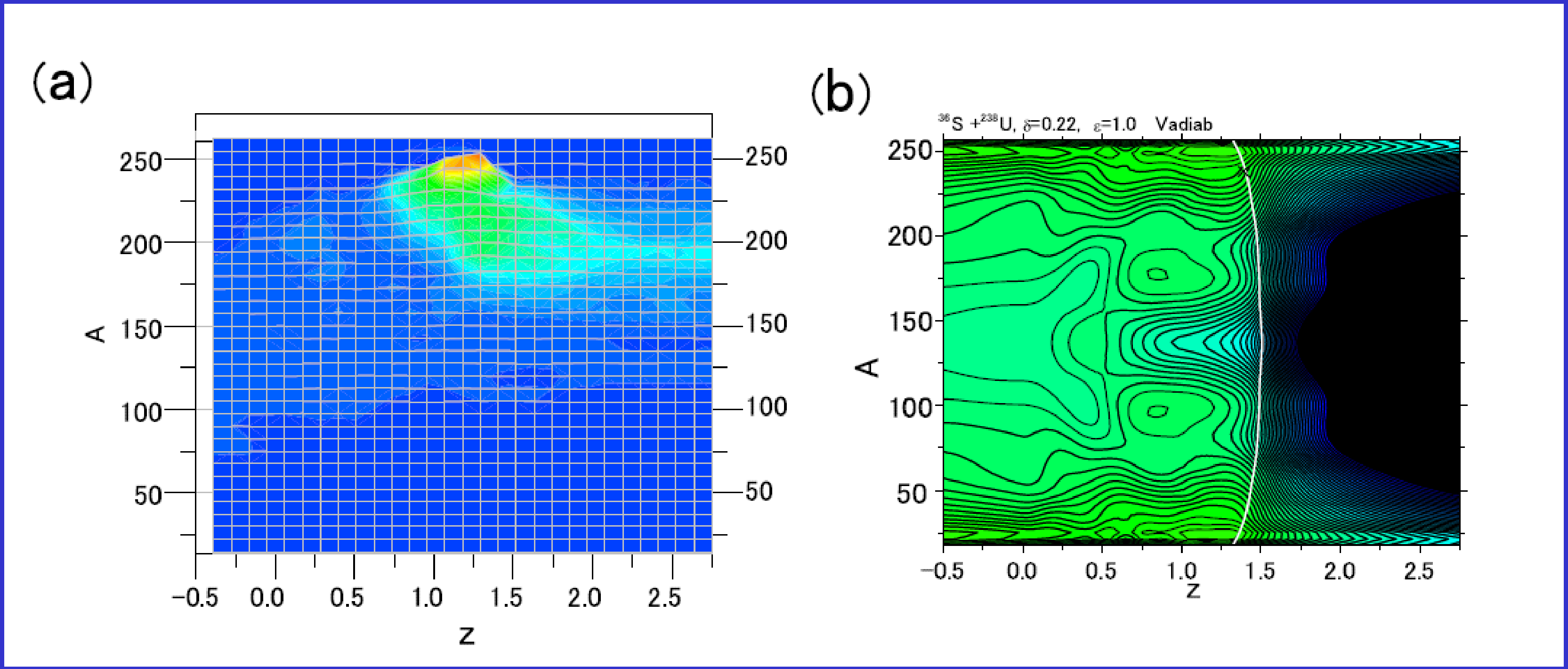


E^* =
35.5 MeV
L=0, θ=0

$\delta = 0.22$
$\varepsilon = 1.0$

$\delta = 0.22$
$\varepsilon = 0.35$

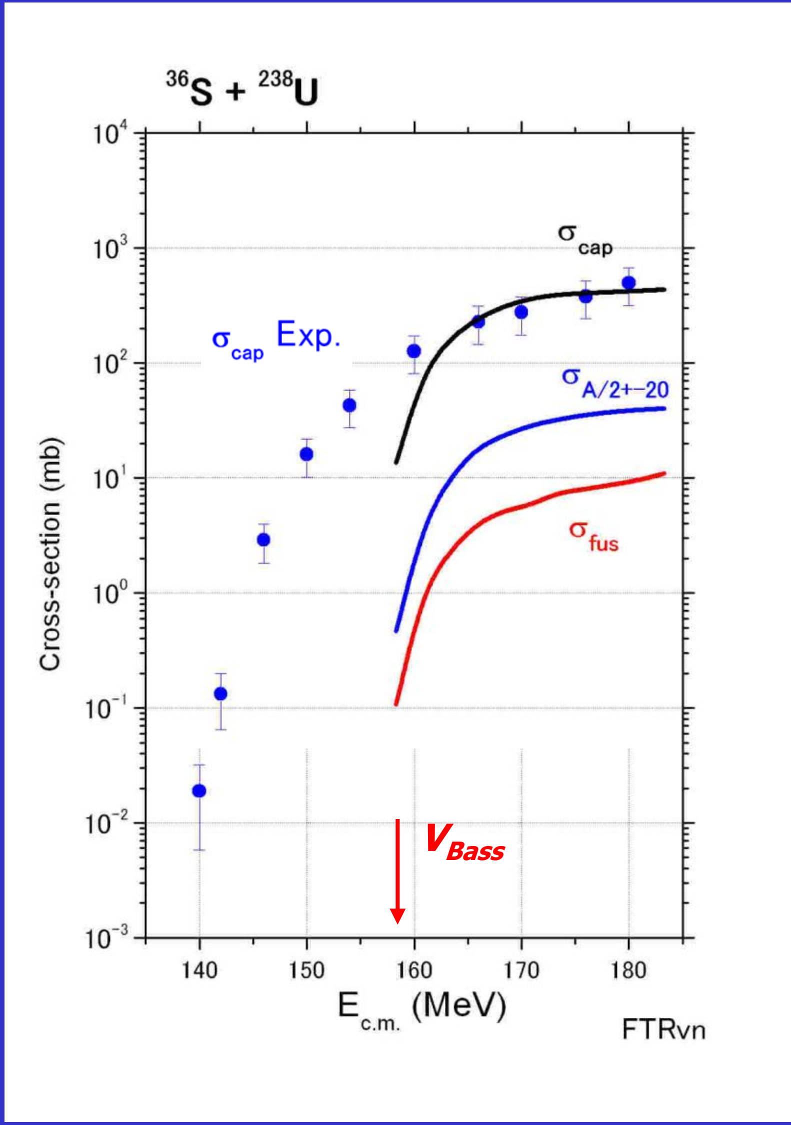
Probability distribution $^{36}S+^{238}U$ on the z-A plane

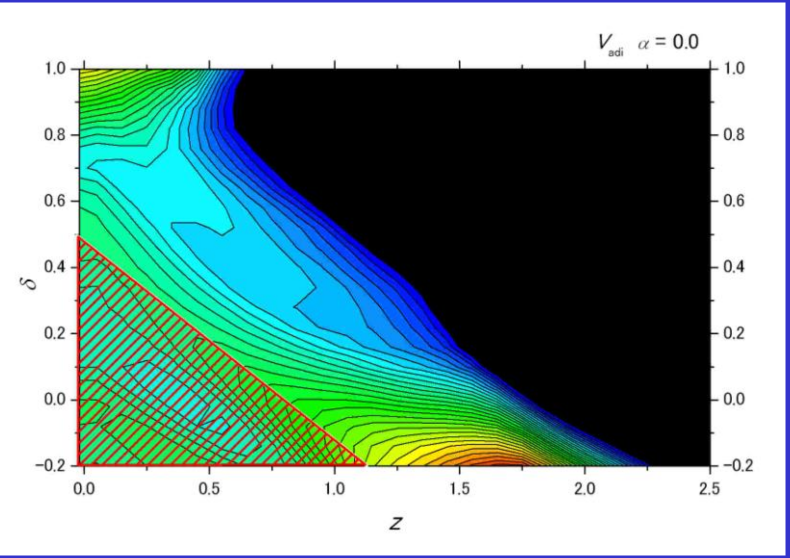


*E** =
39.5 MeV
L=0, θ=0

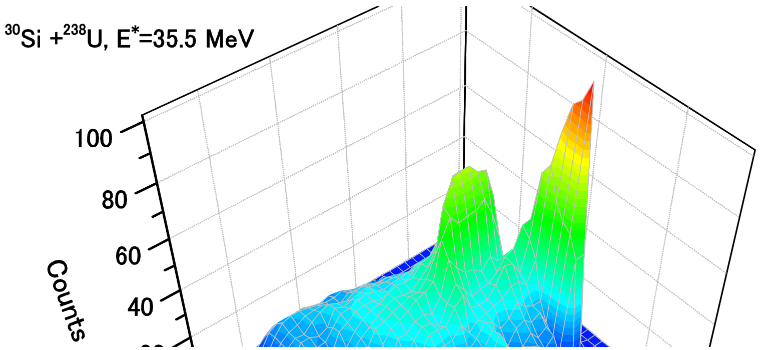
δ = 0.22
ε = 1.0

Cross section

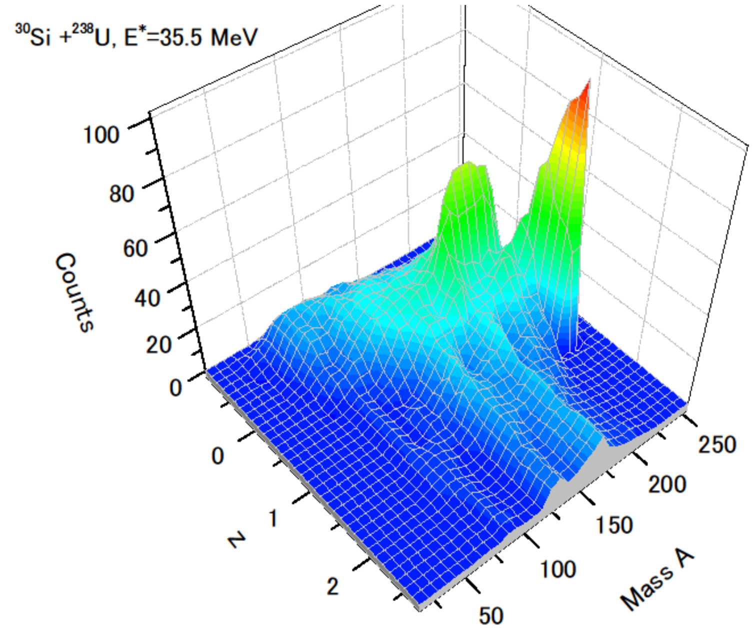




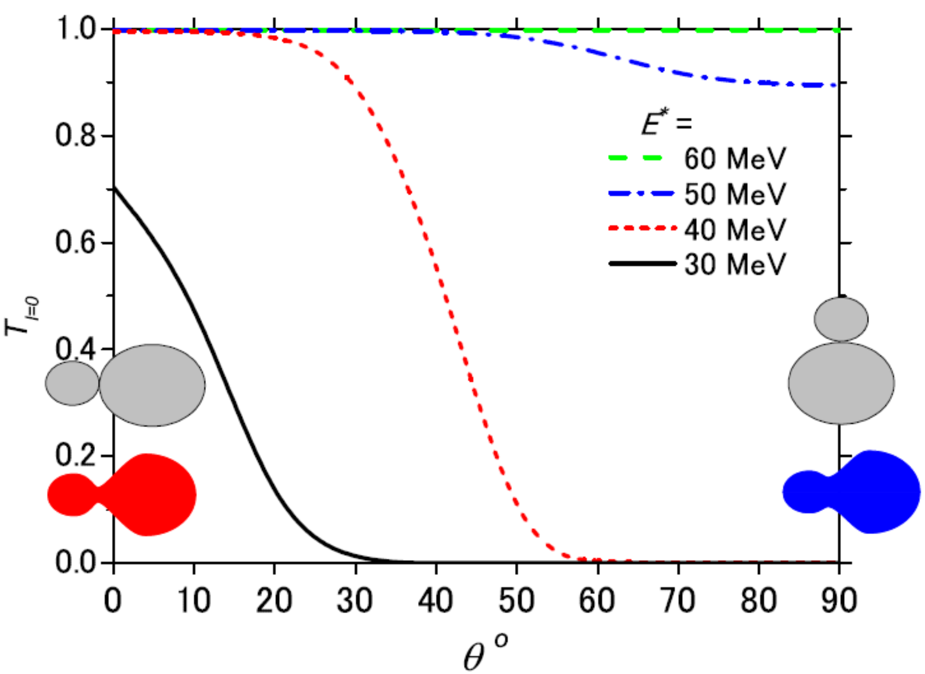
Exp. by K. Nishio *et al*., Phys. Rev. C, **77** (2008) 064607.



30Si +238U, E*=35.5 MeV
100
80
60
40
Counts



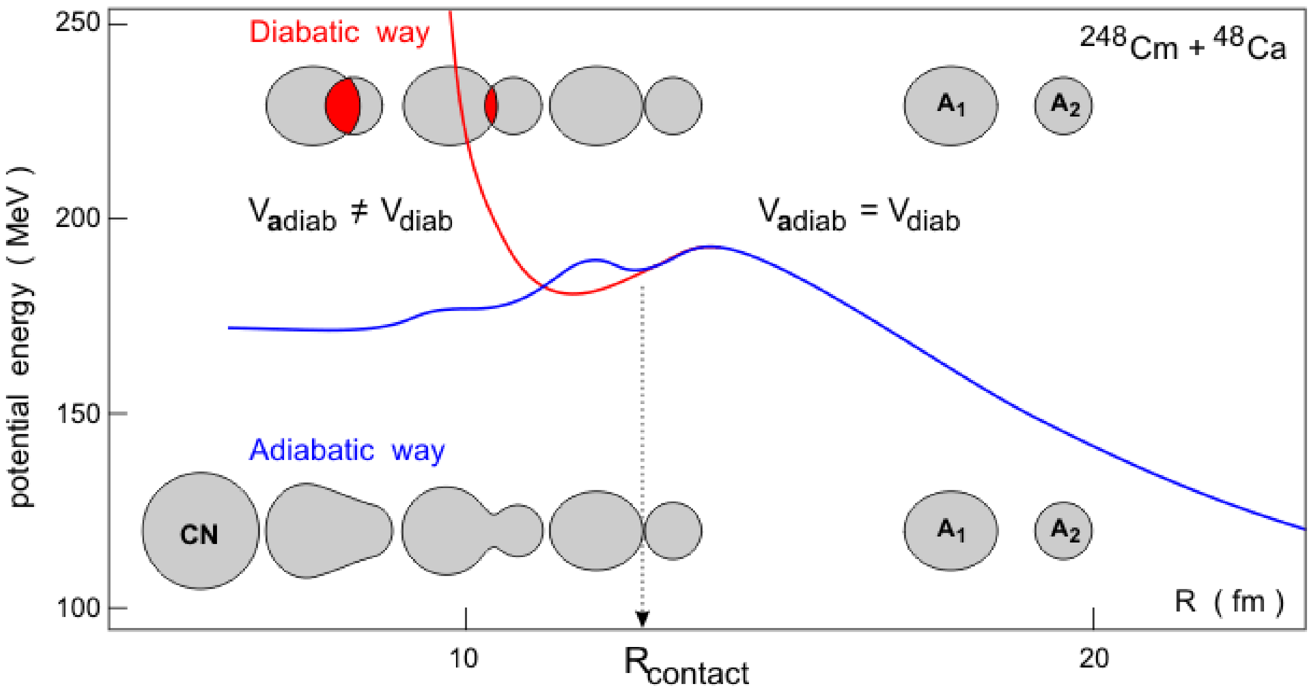
30Si +238U, E*=35.5 MeV
100
80
60
40
20
0
Counts
0
1
2
z
50
100
150
200
250
Mass A



1.0
0.8
0.6
0.4
0.2
0.0
T l=0
0
10
20
30
40
50
60
70
80
90
θ °
E* =
60 MeV
50 MeV
40 MeV
30 MeV

Diabatic and Adiabatic Potential Energy

$$V_{diabat}(R,\beta_1,\beta_2,\alpha,...) = V_{12}^{folding}(Z_1,N_1,Z_2,N_2; R, \beta_1,\beta_2,...) + M(A_1) + M(A_2) - M(Proj) - M(Targ)$$



V. Zagrebaev, A. Karpov, Y. Aritomo, M. Naumenko and W. Greiner, Phys. Part. Nucl. 38 (2007) 469

$$V_{adiabat}(R,\beta_1,\beta_2,\alpha,...) = M_{TCSM}(R,\beta_1,\beta_2,\alpha,...) - M(Proj) - M(Targ)$$

Time-dependent driving potential has to be used

$$V(t) = V_{diab}(\xi)\cdot exp(-\frac{t_{int}}{\tau_{relax}}) + V_{adiab}(\xi)\cdot[1-exp(-\frac{t_{int}}{\tau_{relax}})]$$

$\tau_{relax} \sim 10^{-21}$ s

Time-dependent weight function

the same degrees of freedom !

G. F. Bertsch, 1978; *W. Cassing, W. Nörenberg*, 1983. *A. Diaz-Torres*, 2004; *A. Diaz-Torres and W. Scheid*, 2005.

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System of coupled Langevin type Equations of Motion

$$\frac{dR}{dt} = \frac{p_R}{\mu_R}$$

Variables: $\{R, \theta, \varphi_1, \varphi_2, \beta_1, \beta_2, \eta\}$

$$\frac{d\vartheta}{dt} = \frac{\ell}{\mu_R R^2}$$

$$\frac{d\varphi_1}{dt} = \frac{L_1}{\Im_1}, \quad \frac{d\varphi_2}{dt} = \frac{L_2}{\Im_2}$$

$$\frac{d\beta_1}{dt} = \frac{p_{\beta 1}}{\mu_{\beta 1}}$$

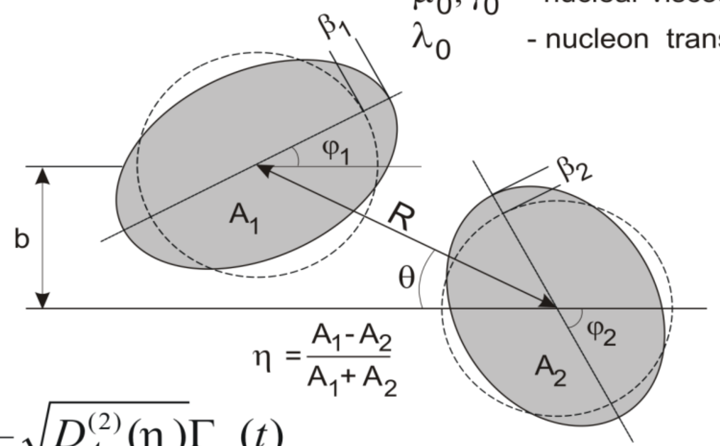
$$\frac{d\beta_2}{dt} = \frac{p_{\beta 2}}{\mu_{\beta 2}}$$

$$\frac{d\eta}{dt} = \frac{2}{A_{CN}} D_A^{(1)}(\eta) + \frac{2}{A_{CN}} \sqrt{D_A^{(2)}(\eta)}\,\Gamma_\eta(t)$$

Most uncertain parameters:

μ_0, γ_0 - nuclear viscosity and friction,

λ_0 - nucleon transfer rate



$$\frac{dp_R}{dt} = -\frac{\partial V}{\partial R} + \frac{\ell^2}{\mu_R R^3} + \left(\frac{\ell^2}{2\mu_R^2 R^2} + \frac{p_R^2}{2\mu_R^2}\right)\frac{\partial \mu_R}{\partial R} + \frac{p_{\beta 1}^2}{2\mu_{\beta 1}^2}\frac{\partial \mu_{\beta 1}}{\partial R} + \frac{p_{\beta 2}^2}{2\mu_{\beta 2}^2}\frac{\partial \mu_{\beta 2}}{\partial R} - \gamma_R \frac{p_R}{\mu_R} + \sqrt{\gamma_R T}\,\Gamma_R(t)$$

$$\frac{d\ell}{dt} = -\frac{\partial V}{\partial \vartheta} - \gamma_{\text{tang}}\left(\frac{\ell}{\mu_R R} - \frac{L_1}{\Im_1} a_1 - \frac{L_2}{\Im_2} a_2\right) R + \sqrt{\gamma_{\text{tang}} T}\,\Gamma_{\text{tang}}(t)$$

$$\frac{dL_1}{dt} = -\frac{\partial V}{\partial \varphi_1} + \gamma_{\text{tang}}\left(\frac{\ell}{\mu_R R} - \frac{L_1}{\Im_1} a_1 - \frac{L_2}{\Im_2} a_2\right) a_1 - \frac{a_1}{R}\sqrt{\gamma_{\text{tang}} T}\,\Gamma_{\text{tang}}(t)$$

$$\frac{dL_2}{dt} = -\frac{\partial V}{\partial \varphi_2} + \gamma_{\tan}\left(\frac{\ell}{\mu_R R} - \frac{L_1}{\Im_1} a_1 - \frac{L_2}{\Im_2} a_2\right) a_2 - \frac{a_2}{R}\sqrt{\gamma_{\text{tang}} T}\,\Gamma_{\text{tang}}(t)$$

$$\frac{dp_{\beta 1}}{dt} = -\frac{\partial V}{\partial \beta_1} + \frac{p_{\beta 1}^2}{2\mu_{\beta 1}^2}\frac{\partial \mu_{\beta 1}}{\partial \beta_1} + \frac{p_{\beta 2}^2}{2\mu_{\beta 2}^2}\frac{\partial \mu_{\beta 2}}{\partial \beta_1} + \left(\frac{\ell^2}{2\mu_R^2 R^2} + \frac{p_R^2}{2\mu_R^2}\right)\frac{\partial \mu_R}{\partial \beta_1} - \gamma_\beta \frac{p_{\beta 1}}{\mu_{\beta 1}} + \sqrt{\gamma_{\beta 1} T}\,\Gamma_{\beta 1}(t)$$

$$\frac{dp_{\beta 2}}{dt} = -\frac{\partial V}{\partial \beta_2} + \frac{p_{\beta 1}^2}{2\mu_{\beta 1}^2}\frac{\partial \mu_{\beta 1}}{\partial \beta_2} + \frac{p_{\beta 2}^2}{2\mu_{\beta 2}^2}\frac{\partial \mu_{\beta 2}}{\partial \beta_2} + \left(\frac{\ell^2}{2\mu_R^2 R^2} + \frac{p_R^2}{2\mu_R^2}\right)\frac{\partial \mu_R}{\partial \beta_2} - \gamma_\beta \frac{p_{\beta 2}}{\mu_{\beta 2}} + \sqrt{\gamma_{\beta 2} T}\,\Gamma_{\beta 2}(t)$$

The problem of mass exchange

Variables: elongation R

deformations β_1, β_2

mass asymmetry η

Schrödinger, Newtonian, Langevin type

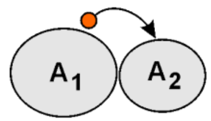
No problems with inertia parameters $M_R(R,\beta,\eta)$, $M_\beta(R,\beta,\eta)$ (Werner - Wheeler)

$$\rightarrow \begin{cases} \frac{dR}{dt} = \frac{p_R}{M_R} \\ \frac{dp_R}{dt} = -\frac{\partial}{\partial R} V(R,\beta,\eta) + \ldots \\ \frac{d\beta}{dt} = \frac{p_\beta}{M} \\ \frac{dp_\beta}{dt} = -\frac{\partial}{\partial \beta} V(R,\beta,\eta) + \ldots \end{cases}$$

? mass-asymmetry η :

1) discrete nature

2) M_η ($R \gtrsim R_{contact}$) $\rightarrow \infty$



$A_1 \rightarrow A_1 - 1$

$A_2 \rightarrow A_2 + 1$

(L. Moretto, 1974)

Distribution function $\varphi(A,t)$ → Master equation

$$\frac{\partial \varphi}{\partial t} = \sum_{A'=A\pm 1} \lambda(A' \rightarrow A)\cdot\varphi(A') - \lambda(A \rightarrow A')\cdot\varphi(A)$$

$$\frac{\partial \varphi}{\partial t} = -\frac{\partial}{\partial A}\left(D^{(1)}\varphi\right) + \frac{\partial^2}{\partial A^2}\left(D^{(2)}\varphi\right)$$

Fokker - Planck (W. Nörenberg, 1974)

$$\frac{dA}{dt} = D^{(1)} + \sqrt{D^{(2)}}\,\Gamma(t)$$

Langevin type eq.

$$\eta = \frac{A_1 - A_2}{A_{CN}}$$

$$\frac{d\eta}{dt} = \frac{2}{A_{CN}} D_A^{(1)}(\eta) + \frac{2}{A_{CN}}\sqrt{D_A^{(2)}(\eta)}\,\Gamma(t)$$

$$D^{(1)} = \int dA'(A'-A)\lambda(A \rightarrow A')$$

$$D^{(2)} = \frac{1}{2}\int dA'(A'-A)^2\lambda(A \rightarrow A')$$

at A = A ± 1

$$D^{(1)} = \lambda(A \rightarrow A+1) - \lambda(A \rightarrow A-1)$$

$$D^{(2)} = \frac{1}{2}[\lambda(A \rightarrow A+1) + \lambda(A \rightarrow A-1)]$$

transition probability

$$\lambda^{(\pm)} = \lambda_0 \sqrt{\frac{\rho(A\pm 1)}{\rho(A)}}\, P_{tr}(R; A \rightarrow A\pm 1) \approx \lambda_0 \exp\left(\frac{V(R,\beta,A\pm 1) - V(R,\beta,A)}{2T}\right) P_{tr}(R; A \rightarrow A\pm 1)$$

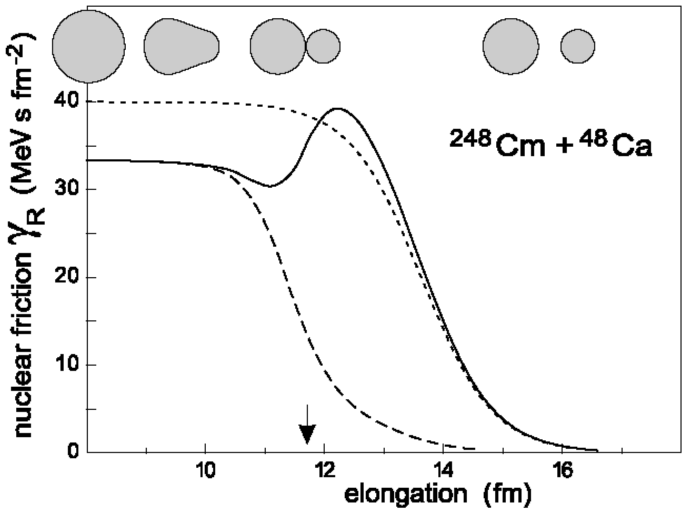
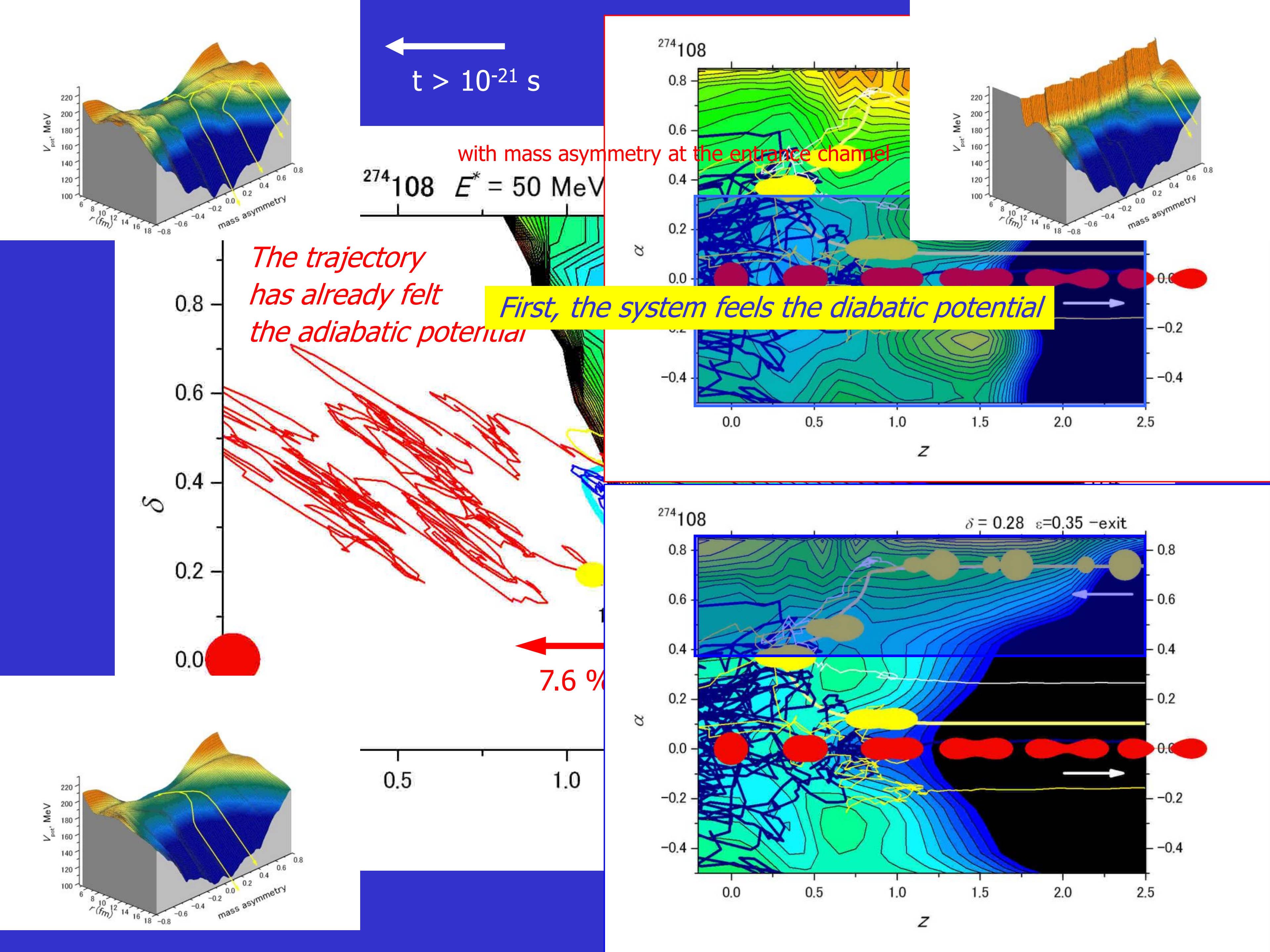


Figure 9. The radial nuclear friction for ^{48}Ca+^{248}Cm collision at zero deformations and fixed mass asymmetry $\alpha = 0.675$. Dotted, dashed and solid curves show the phenomenological friction γ_R^F in the entrance channel at $\gamma_R^0 = 40 \times 10^{-22}$ MeV s fm^{-2}, $\rho_F = 2$ fm and $a_F = 0.6$ fm, the two-body friction γ_R^{WW} for mono-nucleus at $\mu_0 = 3 \times 10^{-23}$ MeV s fm^{-3}, and the resulting friction, respectively. The contact point is indicated by the arrow.

we use here for separated nuclei the phenomenological nuclear friction forces with the Woods–Saxon radial form factor $F(\zeta) = (1 + e^{\zeta})^{-1}$, $\zeta = (\xi - \rho_F)/a_F$. The shift $\rho_F \sim 2$ fm serves to approach the position of the friction shape function to the strong absorption distance which is normally larger than the contact distance R_{contact} [37]. Thus $\gamma_R^F = \gamma_R^0 F(\xi - \rho_F)$, $\gamma_{\mathrm{tang}}^F = \gamma_t^0 F(\xi - \rho_F)$ and γ_R^0, γ_t^0, ρ_F and $a_F \sim 0.6$ fm are the model parameters.

V.I. Zagrebaev and W. Greiner, J. Phys. G. 31 (2005) 825


t > 10-21 s
with mass asymmetry at the entrance channel
274108 E* = 50 MeV
The trajectory
has already felt
the adiabatic potential
First, the system feels the diabatic potential
7.6 %
δ = 0.28 ε=0.35 −exit
mass asymmetry
r (fm)
V_pot, MeV
δ
α
z

Langevin type equation

Before touching nucleon transfer

$$\frac{d\alpha}{dt} = \frac{2}{A_{CN}} D_A^{(1)}(\alpha) + \frac{2}{A_{CN}} \sqrt{D_A^{(2)}(\alpha)\Gamma_\alpha(t)}$$

$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j$$

$$\frac{d\vartheta}{dt} = \frac{\ell}{\mu_R R^2}$$

$$\frac{d\varphi_1}{dt} = \frac{L_1}{\Im_1}$$

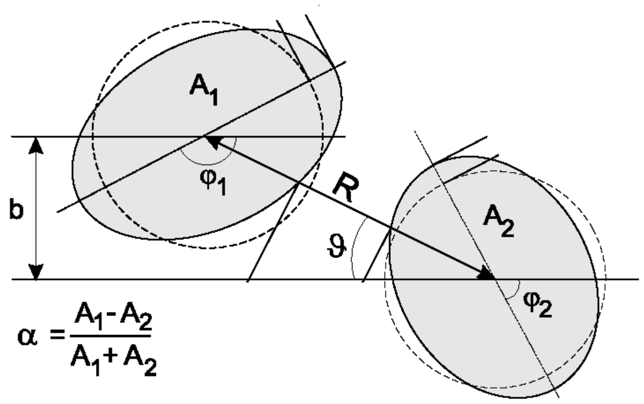
$$\frac{d\varphi_2}{dt} = \frac{L_2}{\Im_2}$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{jk} p_j p_k - \gamma_{ij}\left(m^{-1}\right)_{jk} p_k + g_{ij} R_j(t)$$

$$\frac{d\ell}{dt} = -\frac{\partial V}{\partial \vartheta} - \gamma_{\tan g}\left(\frac{\ell}{\mu_R R} - \frac{L_1}{\Im_1}a_1 - \frac{L_2}{\Im_2}a_2\right)R + R g_{\tan g} R_{\tan g}(t)$$

$$\frac{dL_1}{dt} = -\frac{\partial V}{\partial \varphi_1} + \gamma_{\tan g}\left(\frac{\ell}{\mu_R R} - \frac{L_1}{\Im_1}a_1 - \frac{L_2}{\Im_2}a_2\right)a_1 - a_1 g_{\tan g} R_{\tan g}(t)$$

$$\frac{dL_2}{dt} = -\frac{\partial V}{\partial \varphi_2} + \gamma_{\tan g}\left(\frac{\ell}{\mu_R R} - \frac{L_1}{\Im_1}a_1 - \frac{L_2}{\Im_2}a_2\right)a_2 - a_2 g_{\tan g} R_{\tan g}(t)$$



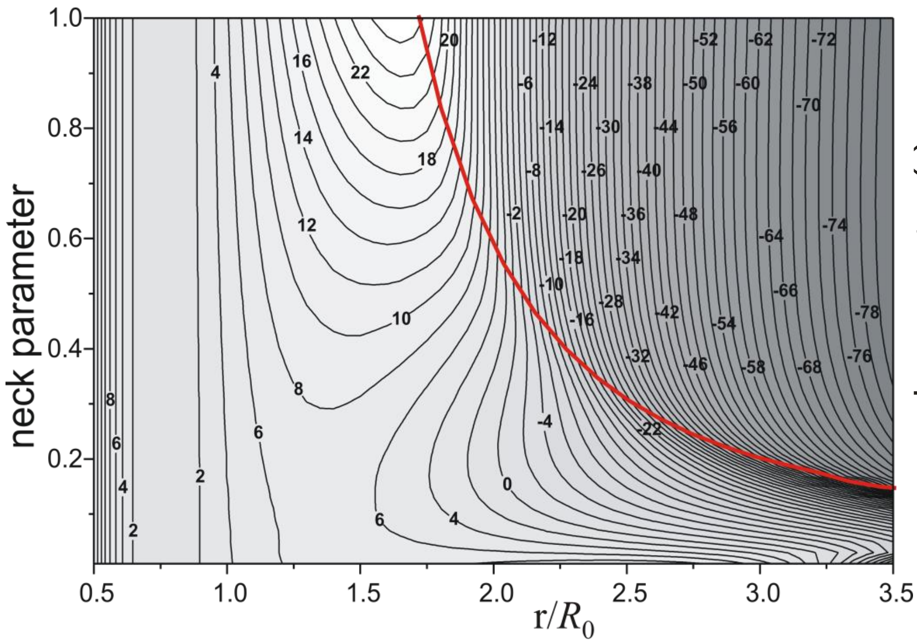
$$\alpha = \frac{A_1 - A_2}{A_1 + A_2}$$

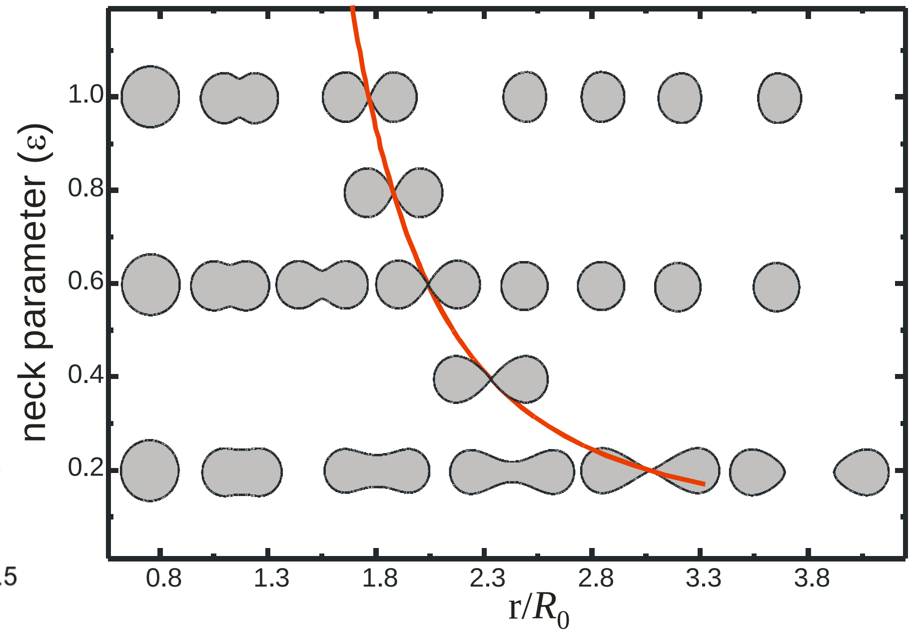
m_{ij} : Hydrodynamical mass (mono-nucleus region), Reduced mass (separated region)

γ_{ij}: Wall and Window (one-body) dissipation

Time dependent adiabatic fusion-fission potential

^{224}Th





$$V_{\text{adiab}}(r,\delta,\alpha,\varepsilon;t)=V_{\text{adiab}}(r,\delta,\alpha,\varepsilon=1)\cdot\exp\left(-\frac{t}{\tau_\varepsilon}\right)+V_{\text{adiab}}(r,\delta,\alpha,\varepsilon=\varepsilon_{out})\cdot\left[1-\exp\left(-\frac{t}{\tau_\varepsilon}\right)\right]$$

V. Zagrebaev, A. Karpov, Y. Aritomo, M. Naumenko and W. Greiner, Phys. Part. Nucl. 38 (2007) 469

$$\tau_\varepsilon = 10^{-20}\ \text{sec}$$

Time-dependent weight function

$$\sigma_{\text{cap}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{cap}}(E;\theta),$$

$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;$$

$$\sigma_{\text{cap}}(E;\theta) = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)T_\ell(E;\theta),$$

$$\sigma_{\text{fus}}(E;\theta) = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)T_\ell(E;$$

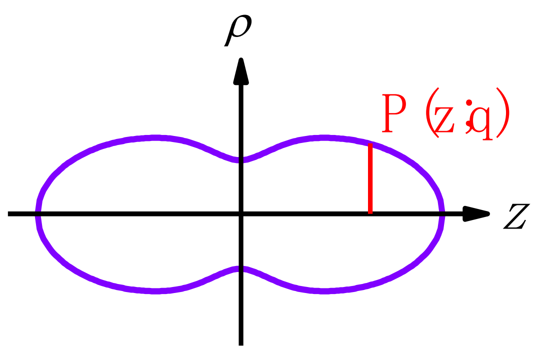
$$\begin{aligned} V(r,\theta) &= V_N(r,\theta) + V_C(r,\theta), \\ V_N(r,\theta) &= \frac{-V_0}{1+\exp[(r-R-R_T\beta_2 Y_{20}(\theta) - R_T\beta_4 Y_{40}(\theta))/a]}, \\ V_C(r,\theta) &= \frac{Z_P Z_T e^2}{r} \\ &\quad + \sum_\lambda \left(\beta_\lambda + \frac{2}{7}\sqrt{\frac{5}{\pi}}\beta_2^2\delta_{\lambda,2}\right)\frac{3Z_P Z_T e^2}{2\lambda+1}\frac{R_T^\lambda}{r^{\lambda+1}}Y_{\lambda 0}(\theta). \end{aligned}$$

Transport coefficients (inertia mass and friction)

Inertia Mass (Hydrodynamical mass)

Total kinetic energy of system

$$T = \frac{1}{2}\rho_m \int v^2 d^3 r = \frac{1}{2}\sum m_{ij}(q)\dot{q}_i \dot{q}_j$$



Werner-Wheeler approximation

$\nabla \cdot \vec{v} = 0$ Incompressible fluid

$\vec{v} = \dot{\rho}\vec{e}_\rho + \dot{z}\vec{e}_z$ Axially symmetric shape

$$\dot{z} = \sum A_i(z;q)\dot{q}_i$$

$$\dot{\rho} = \frac{\rho}{P}\sum B_i(z;q)\dot{q}_i$$

$$P = P(z;q)$$

$$m_{ij} = \pi\rho_m \int_{z_{\min}}^{z_{\max}} P^2 \left(A_i A_j + \frac{1}{8} P^2 A_i' A_j' \right) dz$$

For an incompressible fluid the total (convective) time derivative of any fluid volume must vanish

$$A_i(z;q) = \frac{1}{P^2(z;q)} \frac{\partial}{\partial q_i} \int_z^{z_{\max}} P^2(z';q) dz'$$

$$A_i(z;q) = -\frac{1}{P^2(z;q)} \frac{\partial}{\partial q_i} \int_{z_{\min}}^{z} P^2(z';q) dz'$$

$$B_i(z;q) = -\frac{1}{2} P \frac{\partial A_i}{\partial z}$$

K.T.R. Davies, A.J. Sierk, R. Nix, PRC 13 (1976) 2385

Transport coefficients (inertia mass and friction)

Friction (Two body friction) Incompressible fluid

Rayleigh dissipation function

Total kinetic energy of system

$$F = \frac{1}{2}\mu\int \Phi(r) d^3r = \frac{1}{2}\sum \eta_{ij}(q)\dot{q}_i\dot{q}_j \longleftrightarrow T = \frac{1}{2}\rho_m \int v^2 d^3r = \frac{1}{2}\sum m_{ij}(q)\dot{q}_i\dot{q}_j$$

$$\Phi(r) = \nabla^2 v^2 + \omega^2 - 2\nabla(\vec{v}\times\vec{\omega})$$

$$\vec{\omega} = \nabla\times\vec{v}$$

μ Constant two-body viscosity coefficient

Two body friction

$$\eta_{ij} = \pi\mu\int_{z_{\min}}^{z_{\max}} P^2\left(3A_i'A_j' + \frac{1}{8}P^2 A_i''A_j''\right)dz$$

$$A_i(z;q) = \frac{1}{P^2(z;q)}\frac{\partial}{\partial q_i}\int_z^{z_{\max}} P^2(z';q)dz'$$

$$A_i(z;q) = -\frac{1}{P^2(z;q)}\frac{\partial}{\partial q_i}\int_{z_{\min}}^{z} P^2(z';q)dz'$$

$$B_i(z;q) = -\frac{1}{2}P\frac{\partial A_i}{\partial z}$$

Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \frac{\partial F}{\partial \dot{q}_i}$$

K.T.R. Davies, A.J. Sierk, R. Nix, PRC 13 (1976) 2385

Transport coefficients (inertia mass and friction)

Friction (One body friction)

Rayleigh dissipation function

$$F = \frac{1}{2}\frac{dE}{dt} = \frac{1}{2}\sum \gamma_{ij}(q)\dot{q}_i\dot{q}_j \quad \longleftrightarrow \quad F = \frac{1}{2}\mu\int \Phi(r)d^3r = \frac{1}{2}\sum \eta_{ij}(q)\dot{q}_i\dot{q}_j$$

Incompressible fluid
constant two-body viscosity coefficient

Loss of energy to particles inside the mean filed at the rate

$$\frac{dE}{dt} = \rho_S\bar{v}\int \dot{n}^2 dS$$

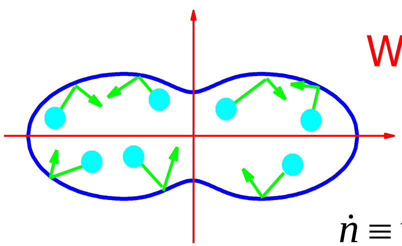
$\rho_S = \rho_S(q,z)$ mass density of nucleus

$\bar{v}$ average nucleon speed

$\dot{n}$ relative normal velocity of the wall

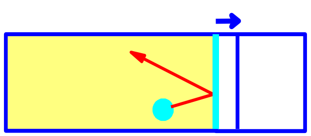
Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \frac{\partial F}{\partial \dot{q}_i}$$



Wall formula

$$\dot{n} \equiv \vec{v}\cdot\hat{n} = \frac{\partial \rho_S}{\partial t}\left[1 + \left(\frac{\partial \rho_S}{\partial z}\right)^2\right]^{-\frac{1}{2}}$$



$$= \sum_i \dot{q}_i \rho_S \frac{\partial \rho_S}{\partial q_i}\left[\rho_S^2 + \left(\rho_S\frac{\partial \rho_S}{\partial z}\right)^2\right]^{-\frac{1}{2}}$$

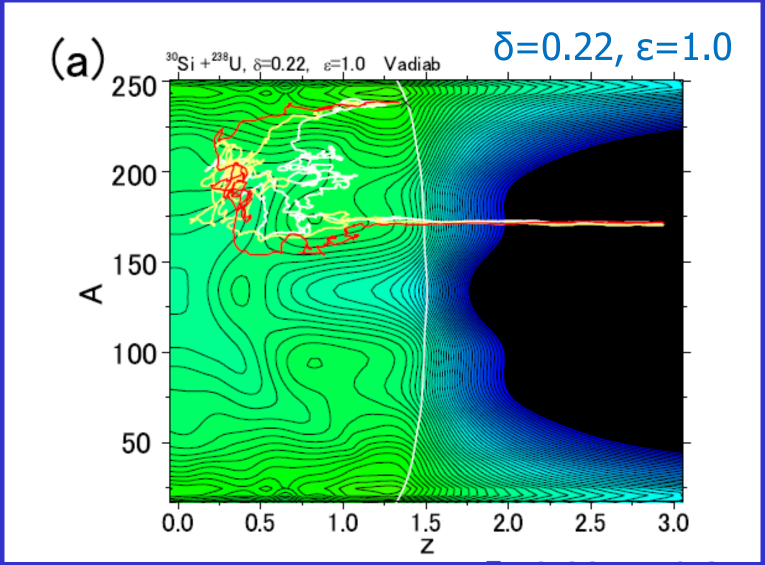
$$\gamma_{ij} = \frac{\pi\rho\bar{v}}{2}\int_{z_{\min}}^{z_{\max}} dz\, \frac{\partial \rho_S^2}{\partial q_i}\frac{\partial \rho_S^2}{\partial q_j}\left[\rho_S^2 + \frac{1}{4}\left(\frac{\partial \rho_S^2}{\partial z}\right)^2\right]^{-\frac{1}{2}}$$

One body friction (Wall formula)

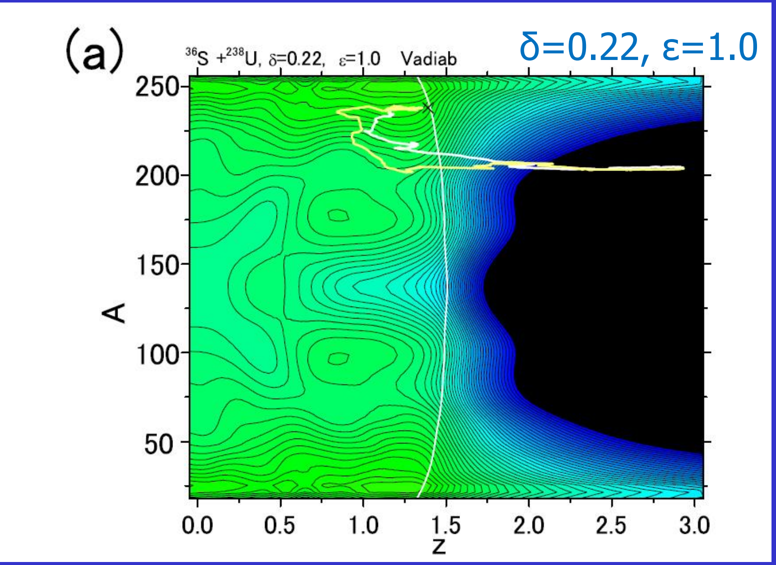
A.J. Sierk, R. Nix, PRC 21 (1980) 982

(b) Trajectory Analysis on Potential Energy Surface z-A plane

$^{30}Si+^{238}U$ E^* = 35.5 MeV
L=0, θ=0



$^{36}S+^{238}U$ E^* = 39.5 MeV
L=0, θ=0



(a) 1-dim Potential energy on scission line